MASTER IN
DATA INTELLIGENCE & STRATEGIE DECISIONALI

SAPIENZA
UNIVERSITÀ DI ROMA

10^{th} RAMOO Workshop

Recent Advances in Multi-Objective Optimization

Sapienza University of Rome, 14^{th} September 2023

Book of Abstracts

The Organizing Committee

Lavinia Amorosi, Sapienza Università di Roma

Marianna De Santis, Sapienza Università di Roma

Keynote speakers

Gabriele Eichfelder, Uncertain multi-objective optimization and optimization with many objectives

Matthias Ehrgott, Walking School Bus Line Routing for Efficiency, Health and Walkability: A Multi-objective Optimisation Approach

Uncertain multi-objective optimization and optimization with many objectives

Gabriele Eichfelder
Fakultät Mathematik und Naturwissenschaften, Technische Universität Ilmenau

In many applications one has to deal with various difficulties at the same time, like uncertain data or several competing objective functions. For instance, for the integration of neighborhood networks into overarching distributing energy networks, a pure optimization of the neighborhood networks under an externally defined weighting of the relevant targets does not adequately model the problem. Moreover, uncertainties in the form of fluctuations or other disturbances can appear and a found solution has to be robust against that. A robust approach to uncertain multiobjective optimization corresponds to solving a set-valued optimization problem. However, it is a very difficult task to solve these optimization problems even for specific cases. In this talk, we present a parametric multiobjective optimization problem for which the optimal solutions are strongly related to the robust solutions of the uncertain multiobjective problem. With this approach we can approximate the robust solution set with desired accuracy. However, this comes at a price: the parametric multiobjective problem has many objective functions with symmetries in the optimal solution set. This motivates the need of developing numerical solution approaches for many objective optimization problems.

Walking School Bus Line Routing for Efficiency, Health and Walkability: A Multi-objective Optimisation Approach

Matthias Ehrgott
Department of Management Science, Lancaster University

joint work with: Judith Y.T. Wang, Zhengyu Wu, Yating Kang, Mengfan Wen, Christopher Rushton.

Walking School Bus (WSB) has been recognised as an innovative solution to promote walking to school, bringing a wide spectrum of benefits, including: health benefits from the physical exercise, social skills and traffic reduction. To facilitate the success of WSBs, one vital element is its route planning, which directly affects the catchment for the service and the realisation of all the potential benefits. Previously, time has been the only factor that has been considered in WSB routing problems. Other important factors including air quality, safety and comfort will also be considered in this paper. Air quality along a WSB route is important to help realise the health benefits of walking. Traffic safety has been the biggest barrier to walking to school and must be addressed in planning a WSB route. Ensuring children have an enjoyable and comfortable experience is vital for the sustainability and success of WSB.

A walking network is introduced to enable modelling pedestrian movements in detail, including walking movements on different sides of the road and crossing movements. This approach enables detailed route-based analysis to assess the localised effect of air quality on pollutant dose. We define walkability as a measure of children's needs in safety and comfort, which can also be assessed in detail on each route. We propose a multi-objective optimisation model to generate efficient WSB routes with three objectives representing the potential benefits of WSB: (1) to minimise time; (2) to minimise pollutant dose; and (3) to maximise walkability. We apply our model to a selected school in Bradford in the UK, generating three WSB lines following *efficient* routes. These lines go through a predetermined sequence of "WSB Stops", with the final stop as the school. All children within the catchment area will be able to join a WSB within 1-2 minutes' walk from their home to the nearest stop. Our multi-objective WSB route planning model is highly transferable to any selected school in any WSB targeted area. Planners will be able to select a combination of WSB lines to offer, based on the requirement of coverage area and resource availability.

[1] J.Y.T. Wang, Z. Wu, Y. Kang, E. Brown, M. Wen, C. Rushton, M. Ehrgott. Walking School Bus Line Routing for Efficiency, *Health and Walkability: A Multi-objective Optimisation Approach*, Journal of Multi-Criteria Decision Analysis 2023, DOI: 10.1002/mcda.1803

Advances in Continuous Multi-Objective Optimization

Ana Luisa Custodio A trust-region approach for computing Pareto fronts in multiobjective optimization

Pierluigi Mansueto Improved Front Steepest Descent for Multi-objective Optimization

Bennet Gebken Similarities between multi-objective optimization and nonsmooth optimization

A trust-region approach for computing Pareto fronts in multiobjective optimization

Ana Luisa Custodio
NOVA School of Science and Technology (NOVA SST), Universidade Nova de Lisboa

joint work with: Aboozar Mohammadi.

We propose an algorithm based on a trust-region approach for computing an approximation to the complete Pareto front of a given multiobjective optimization problem. The algorithm alternates between two main steps, the extreme point step, that moves towards extreme points of the Pareto front, and a scalarization step, that attempts to reduce the gaps on the Pareto front, by solving an adequate scalarization problem. In any of these two steps, models are built to replace the components of the objective function. We will detail the algorithmic structure, both for derivative-based and derivative-free settings, present some theoretical results regarding convergence and illustrate the good numerical performance of the method.

Improved Front Steepest Descent for Multi-objective Optimization

Pierluigi Mansueto
Dipartimento di Ingegneria dell'Informazione (DINFO), Università di Firenze

In this work, we deal with the Front Steepest Descent algorithm for multi-objective optimization. We point out that the algorithm from the literature is often incapable, by design, of spanning large portions of the Pareto front. We thus introduce some modifications within the algorithm aimed to overcome this significant limitation. We prove that the asymptotic convergence properties of the algorithm are preserved and numerically show that the proposed method significantly outperforms the original one. Lastly, we evidence the effectiveness and efficiency of the new approach in a different harder setting like the cardinality-constrained multi-objective optimization.

Similarities between multi-objective optimization and nonsmooth optimization

Bennet Gebken

Institut für Industriemathematik, Universität Paderborn

At first glance, multi-objective optimization (MOO) and nonsmooth (single-objective) optimization (NSO) are two distinct subclasses of general optimization. But upon closer analysis, it turns out that there are several parallels. When considering first-order information then, in both areas, there is not only one but multiple gradients that have to be considered simultaneously: In MOO, these are the gradients of the different objective functions, and in NSO, these are all the subgradients from the Clarke subdifferential. As such, there are strong structural similarities when considering results like optimality conditions and descent directions. In addition to theoretical results, there are also applications where MOO and NSO naturally meet: In many practical problems, a weighted, nonsmooth regularization term is added to a smooth objective function to enforce additional properties of the solution. For example, a sparse minimizer of a function can be found by adding a weighted L1-norm to the function. By varying the weight (also known as the regularization parameter), minimizers with varying degrees of regularity can be computed. Traditionally, these problems are treated via NSO. But since a regularized objective function can be interpreted as a simple weighted sum scalarization, regularization problems may also be treated via MOO. In this talk, I will present several of these similarities and show how they can be used to obtain new insights and results.

Advances in Discrete Multi-Objective Optimization

Gökhan Kof, Elimination of Objective Functions In Multiobjective Discrete Optimization via Projections

Michael Stiglmayr, An output-polynomial time algorithm to determine all supported efficient solutions for multi-objective integer network flow problems

Elimination of Objective Functions In Multiobjective Discrete Optimization via Projections

Gökhan Kof

Graduate School of Sciences and Engineering, Koç Üniversitesi, Istanbul

joint work with: Serpil Sayın.

In this study, we investigate the consequences of reducing the number of objective functions in.a given multiobjective discrete optimization (MODO) problem. It is well-established that as the number of objectives for a MODO problem increases, the problem becomes harder to solve computationally. If none of the objectives are redundant, ignoring some of them will result in information loss due to missing nondominated points while making the problem computationally easier to solve. We investigate projection based metrics in order to identify which objectives will lead to minimal information loss when removed. We use the similarity between the gradient of an objective function and its projection onto objects defined by the remaining objectives as a proxy for quantifying the contribution of that objective to the diversity of the nondominated solutions in the nondominated set. We assess the effectiveness of the metrics on well-studied problem instances with known nondominated sets. We use quality measures such as coverage error, additive epsilon indicator and hypervolume to evaluate the representations obtained by solving the projected problems. Preliminary findings indicate that proposed projections are effective ways of reducing the number of objectives in order to obtain representations by solving simpler problems.

An output-polynomial time algorithm to determine all supported efficient solutions for multi-objective integer network flow problems

Michael Stiglmayr
Fakultät Mathematik und Naturwissenschaften, Bergische Universität Wuppertal

joint work with: David Könen.

We consider the problem of enumerating all supported efficient solutions for a linear multi-objective integer minimum cost flow problem. Several characterizations for supported nondominated vectors/efficient solutions are used in the literature, which are equivalent in the non-integer case. However, they may lead to different sets of supported nondominated vectors/efficient solutions for MOILPs. This motivates us to distinguish between supported and weakly supported efficient solutions. In this talk we derive an output-polynomial time algorithm to determine all supported efficient solutions for MOIMCF problems. This is the first approach that solves this general problem in output-polynomial time. Moreover, we prove that the existence of an output-polynomial time algorithm to determine all weakly supported nondominated vectors (or all weakly supported efficient solutions) for a MOIMCF problem with three or more objectives can be excluded, unless P = NP.

Tutorial Session

Benjamin Weißing, Tutorial on bensolve tools

Tutorial on bensolve tools

Banjamin Weißing
Facoltà di Economia, Libera Università di Bolzano

In this tutorial we will show how to compute, and apply operations to, convex polyhedra via bensolve tools. This toolbox uses the equivalence between multiple objective linear optimisation and polyhedral projection problems to carry out the manipulations of convex polyhedra. As such it is possible to compute solutions to MOLPs and vector linear programmes (VLPs) with this toolbox. As a use-case, we compute quantiles of a multivariate random variable in the sense of [1] by using its VLP-formulation [2].

1 Andreas H. Hamel, Daniel Kostner, *Cone distribution functions and quantiles for multivariate random variables*, Journal of Multivariate Analysis, Volume 167, 2018, Pages 97-113, ISSN 0047-259X, `https://doi.org/10.1016/j.jmva.2018.04.004.`

2 Andreas Löhne, Benjamin Weißing, *Finite representation of quantile sets for multivariate data via vector linear programming*, arXiv preprint, 2023, `https://doi.org/10.48550/arXiv.2303.15600`

Special Thanks to

Dipartimento di Scienze Statistiche, Sapienza Università di Roma

Paolo Dell'Olmo, Sapienza Università di Roma

Director of Master in "Data intelligence e strategie decisionali"

Isabella Lari, Sapienza Università di Roma

Luís Paquete, Universidade de Coimbra

Sophie Parragh, Johannes Kepler University Linz