Gradient-Based Biobjective Shape Optimization of Ceramic Components: Probabilty of Failure versus Cost

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MATHEMATICAL MODELLING, ANALYSIS AND COMPUTATIONAL MATHEMATICS

BERGISCHE UNIVERSITÄT WUPPERTAL

GIVEN-Project

Shape optimization of gasturbines in volatile energy networks

- Collaboration with the University of Trier
- Project partners are:
  - Siemens AG
  - The German Aerospace Center (DLR)
  - Institute of Material Science at TU Kaiserslautern

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Outline

Plan

Problem

Problem:

- Ceramic component under tensile load
- Optimize its shape
  - improve reliability
  - minimize volume
- Instead of minimizing the peak stress consider the probability of failure

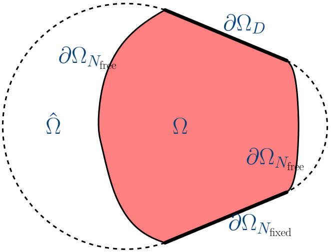
Mathematical Formulation in 2D

$\Omega \subseteq \mathbb{R}^2$: domain with Lipschitz boundary $\partial\Omega$.

$$\partial\Omega = \overline{\partial\Omega}_D \cup \overline{\partial\Omega}_{N_{fixed}} \cup \overline{\partial\Omega}_{N_{free}}.$$

Forces may act on the object with the shape given by Ω.

- Volume force: $\bar{f} \in L^2(\Omega, \mathbb{R}^2)$
- Surface force: $\bar{g} \in L^2(\partial\Omega_N, \mathbb{R}^2)$.



Mathematical Formulation in 2D

- The linear elasticity equation must hold for Ω.

$$B(u,v) = L(v),\ \forall v \in H_0^1(\Omega, \mathbb{R}^2),$$

where

$$\begin{aligned} B(u,v) &= \int_\Omega \sigma(u) : \varepsilon(v)\, dx \\ &= \lambda \int_\Omega \nabla \cdot u \nabla \cdot v\, dx + 2\mu \int_\Omega \varepsilon(u) : \varepsilon(v)\, dx \\ L(v) &= \int_\Omega \bar{f}\, v\, dx + \int_{\partial\Omega_N} \bar{g}\, v\, dA \end{aligned}$$

- Displacement $u \in H^1(\Omega, \mathbb{R}^2)$
- Stress tensor $\sigma(u) = \lambda\, tr(\varepsilon(u))\, I + \mu\, (\varepsilon(u) + \varepsilon(u)^\top)$, strain tensor $\varepsilon(v)$, Lamé's constants λ, μ

Plan

Modeling the Probability of Failure

- Risk of failure arises from flaws already in the material, due to the production process
- Under tensile load, these flaws may become the initial points of a rupture
- Objective function:

$$J(\Omega, Du) := \nu(A_c(\Omega, \nabla u)) = \frac{\Gamma(\frac{d}{2})}{2\pi^{\frac{d}{2}}} \int_{\Omega} \int_{S^{d-1}} \left(\frac{\sigma_n}{\sigma_0}\right)^m dn\, dx$$

Plan

Trade-Off Analysis: Probabilty of Failure versus Cost

$\min$ Probability of Failure (PoF)

$\min$ Volume (Cost)

s.t. State equation and boundary conditions hold

$\rightsquigarrow$ Biobjective optimization problem

Pareto Critical Point

Definition (Pareto Critical Point)

A necessary condition for a point $x \in \mathbb{R}^n$ to be locally Pareto optimal is

$$\{v \in \mathbb{R}^n \,|\, \nabla f_i(x)^\top v < 0, \; \forall\, i = 1, 2\} = \varnothing.$$

If x^* satisfies this condition we call it a *Pareto critical point*.

Plan

Multiobjective Descent Methods

- Fliege and Svaiter (2000):
  Steepest descent methods for multiobjective optimization
- Désidéri (2009, 2012, 2014, ...):
  Multiple-gradient descent algorithm (MGDA)

Fliege and Svaiter, 2000

- Goal: Find a direction $v \in \mathbb{R}^n$ that is a descent direction for *all* objective functions, i.e., $\nabla f_i(x)^\top v < 0, \ i = 1, 2$
- Computing a descent direction $v \in \mathbb{R}^n$:

$$\begin{aligned} \min \quad & \alpha + \frac{1}{2}\|v\|^2 \\ \text{s.t.} \quad & \nabla f_i(x)^\top v \le \alpha, \ i = 1, 2 \end{aligned}$$

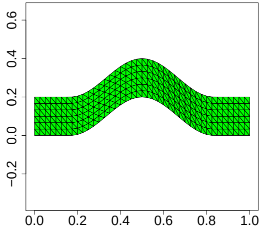
- Step length: Armijo-like rule (componentwise)

Plan

2D Test Case

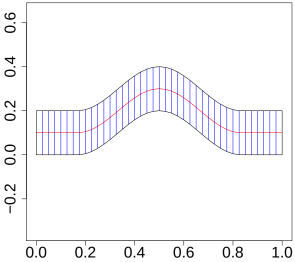
- Beryllium oxide (BeO) with Weibull modulus $m = 5$.
- Testobject:
  - Deformed rod with thickness 0.2m and length 1m
  - Implemented in R
  - Discretized by finite elements (41x7 grid)
  - Objective functions: *probability of failure* and *volume*
  - Boundaries: fix left, pull right

    $\rightsquigarrow$ expected solution: straight rod



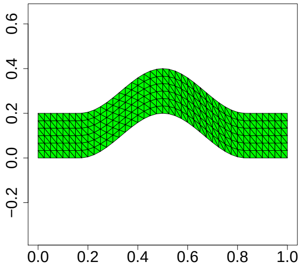
Mesh-Morphing

- Approximation error depends on the mesh-quality
- Restrict movement: fix x coordinates
- Introduce a shape-parameter $\varrho = (\varrho^{ml}, \varrho^{th}) \in \mathbb{R}^{2\cdot 41}$
- Fit via B-splines for implicit smoothening of shapes
  $\rightsquigarrow$ further reduction of variables

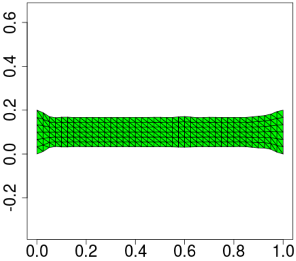


Plan

Shape Optimization

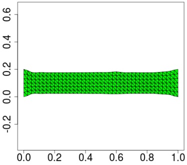


(a) Initial Shape

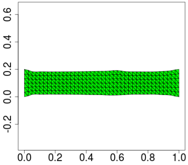


(b) One Optimized Shape

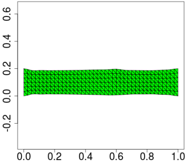
Shape Optimization (More Shapes)



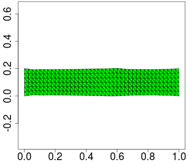
(c)



(d)

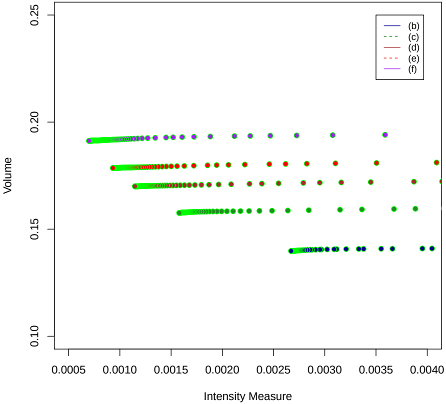


(e)



(f)

Convergence to Pareto Critical Points



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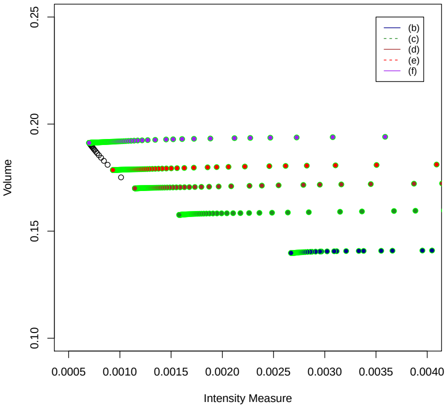

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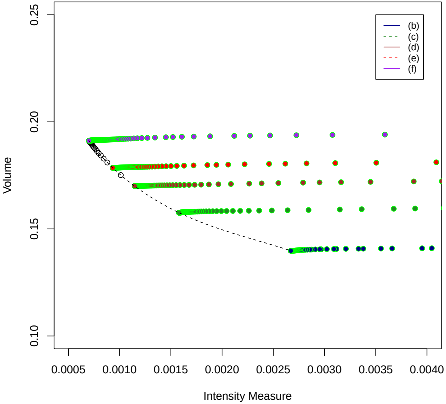

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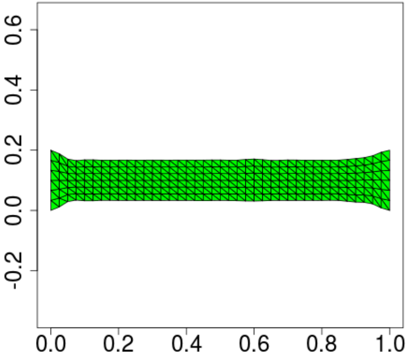

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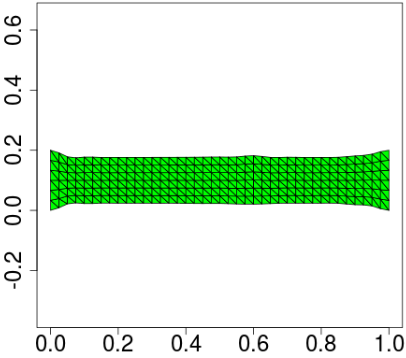

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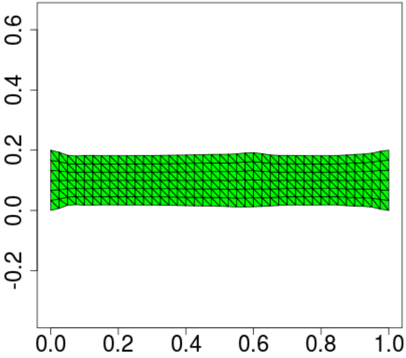

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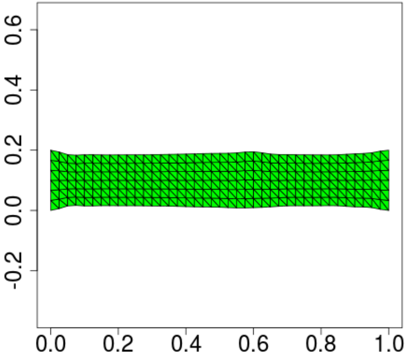

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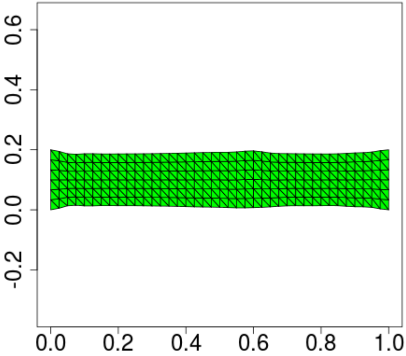

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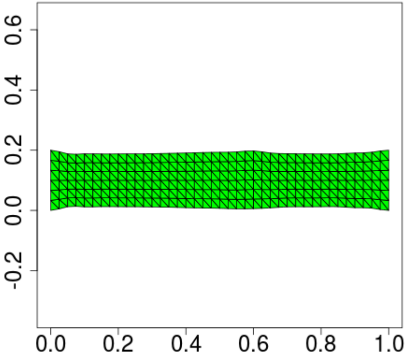

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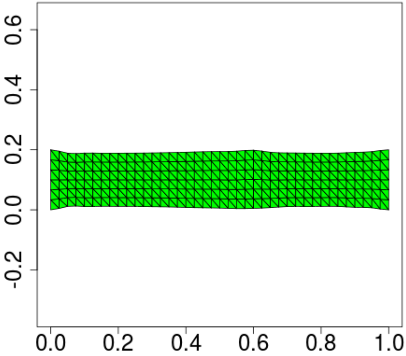

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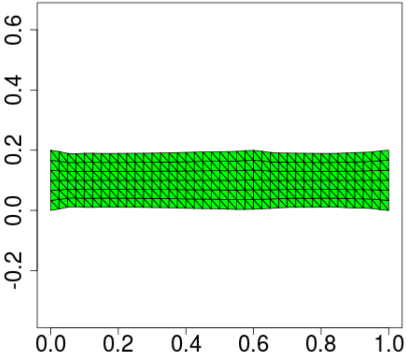

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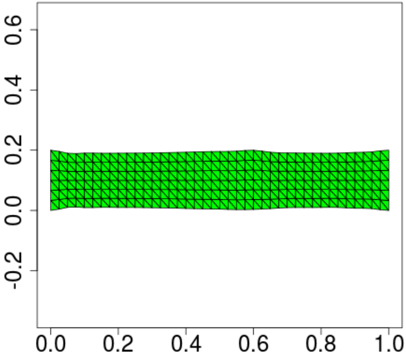

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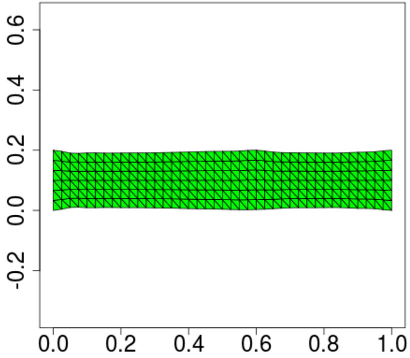

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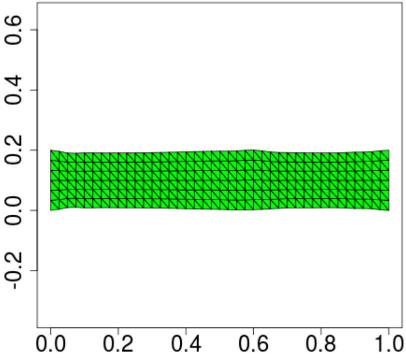

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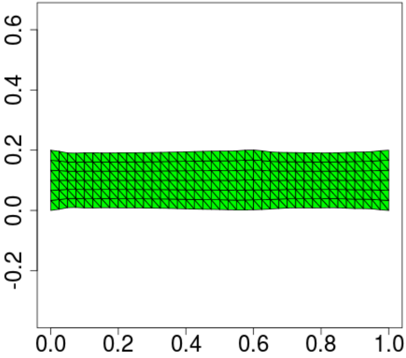

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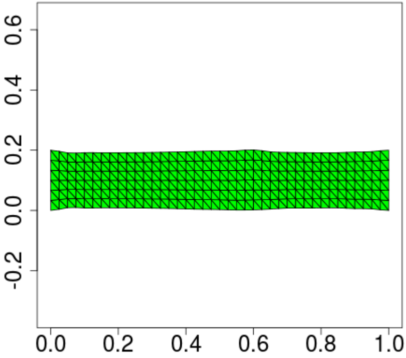

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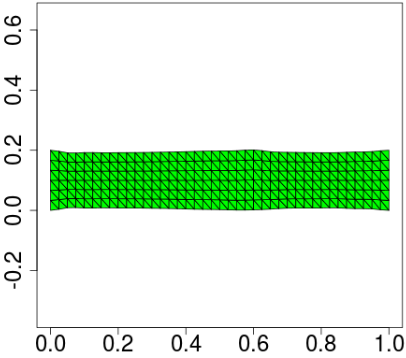

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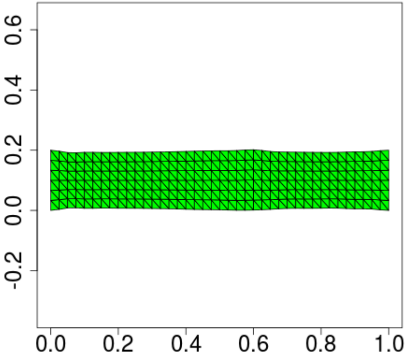

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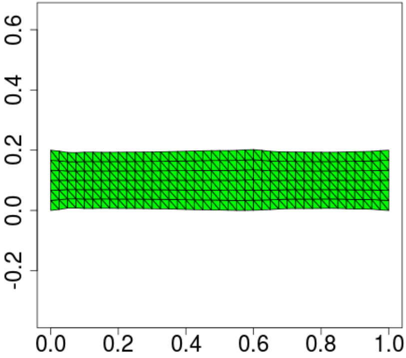
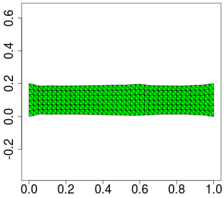


Plan

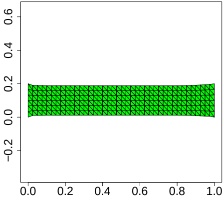
Outlook

- Implementation in Python
- Extension to 3D
- Adaptive mesh refinement
- Avoiding local minima

Comparison with Weighted Sum Approach



(a) B.O.-Descent



(b) Weighted Sum

	(a)	(b)
Iterations	109	133
Int. measure	.00093	.00092
Volume	.17855	.17850

(b) dominates (a)!

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