Scheduling maintenances of nuclear power plants, from 2-stage robust programming to multi-objective optimization

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Aim/highlight of this presentation

- An industrial problem with many types of constraints and variables.
- A 2-stage robust optimization model to fulfill industrial requirements.
- Some hypothesis are different from the state-of-the-art: identifying the methodological and numerical difficulties.
- Bi-objective optimization to overcome these difficulties, and to lead to an industrializable model.

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2-stage Robust model

3 Lighter robustness approach

4 Conclusions and perspectives

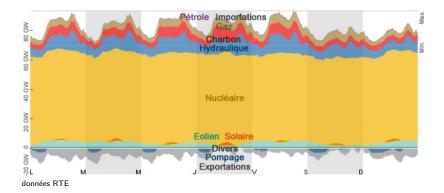
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Problem presentation

- Nuclear reactors must be shut down periodically for maintenance and refueling.
- A two level problem:
 - Main decisions: dates of outages, refueling quantities. Coupling constraints on outages.
 - Second Level: production and stocks variables, to compute the economic cost of the main decisions, fulfilling the technical constraints.
- Stochastic problem modeled for the Challenge EURO/ROADEF 2010.
- Robust extension: maintenance operations are submitted to uncertainty in their durations, try to minimize the impact of these uncertain lateness.

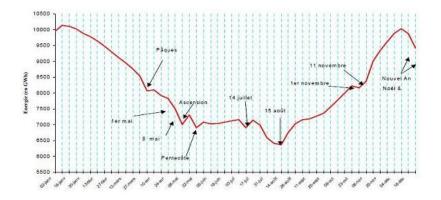
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Power generation in a week



 \Longrightarrow Nuclear power is a "basis" power production mean, few modulation capacities

Power demand in a year



 \implies First fact: trying to place mainly maintenances (and so outages) mainly in Summer periods, avoid Winter periods.

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Problem statement 2-stage Robust model Conclusions and perspectives

Sets and few notations

$j \in \mathcal{J} = \llbracket 1, J rbracket$
$i \in \mathcal{I} = \llbracket 1, I rbracket$
$t \in \mathcal{T} = \llbracket 1, T rbracket$
$w \in \mathcal{W} = \llbracket 1, W \rrbracket$
$k \in \mathcal{K} = \llbracket 0, K \rrbracket$
$s \in \mathcal{S} = \llbracket 1, S rbracket$

Flexible (Type 1, T1) power plants. Nuclear power plants (Type 2, T2). Production time steps, index t corresponds to period [t, t+1]Weekly time steps to place outage dates. Cycles related to T2 units, k = 0 for initial conditions. Stochastic scenarios for demands, production costs and capacities.

Outage duration for maintenance and refueling of T2 unit i at cycle k. Dai.k $\overline{\mathbf{P}}_{i}^{t}$ $\overline{\mathbf{P}}_{jt}^{s}$ Maximal generated power for T2 unit i at time step t.

Maximal generated powers for T1 unit i at scenario s.

N.B: in this talk, we assume $\mathcal{T} = \mathcal{W}$ (aggregating \mathcal{T} time steps to weeks), and only one stochastic scenario (just to avoid a supplementary index, the robust extension applies with several stochastic scenarios)

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Problem Constraints

- Power demand constraints coupling all production units
- Production constraints for generating units:
 - Production bounds for T1 and T2 units
 - Decreasing profile when fuel level of nuclear units is low
- Fuel constraints:
 - Bounds on fuel stocks and refueling levels
 - Maximal threshold of fuel to operate an outage
- Scheduling constraints for outages: resource constraints, different spacing constraints, maximal capacity offline . . .

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Competing approaches for the 2010 ROADEF Challenge

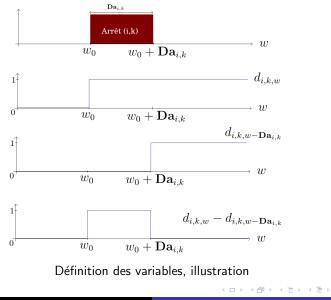
- Best results: simple aggressive and frontal local search heuristics with local moves on outages dates. (Gardi et al and Kuiper et al)
- Heuristics and matheuristics iterating in the natural 2 level structure were not efficient.
- Approaches based on exact methods required model simplifications and reduction sizes(scenarios and time steps aggregated, hierarchical approaches, relaxed constraints and heuristic fixations in postprocessing)
- Only one exact approach did not aggregate the scenarios (Lusby et al for a Benders decomposition approach).

Gardi, F., Nouioua, K.: Local search for mixed-integer nonlinear optimization: a methodology and an application. Lecture Notes in Computer Science 6622, 167–178 (2011) Rozenknopf, A., Calvo, R.W., et al.: Solving the electricity production planning problem by a column generation based heuristic. Journal of Scheduling 16(6), 585–604 (2013) Lusby, R., Muller, L., Petersen, B.: A solution approach based on benders decomposition for the preventive maintenance scheduling problem of a stochastic large-scale energy system. Journal of Scheduling 16(6), 605–628 (2013)

Variable definition

- Binaries: Outage decisions $d_{i,k,w}$ for outage (i, k) and week w: $d_{i,k,w} = 1$ if the outage i, k began before w. (for efficient standard branching)
- Continuous variables:
 - $r_{i,k}$: refueling quantities for the refueling in outage (i, k).
 - $p_{i,k,s,t}$ nuclear (T2) productions T1.
 - $p_{j,s,t}$ non nuclear (T1) productions levels.
 - Residual stocks $x_{i,s}^{fin}$ (for cost function, to avoid end-of-side effects)
- Dependent variables: fuel stocks $x_{i,k,s}^{init}, x_{i,k,s}^{fin}$.

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A glance at the problem structure

$$\min_{\substack{d \in \{0,1\}^n \times \mathbb{R}^m_+, p_s \ge 0 \\ \forall s \quad T \ d + W \ p_s \quad \leqslant h_s \\ \forall s \quad B \ p_s \quad \leqslant b_s}} \sum_{s} c_s \ p_s$$

 $d\colon$ decisions on outages (binaries) and refueling levels (continuous), independent decisions valid for all scenarios (1st stage)

 p_s : production decisions (continuous) and implied fuel levels to have a linear formulation (2nd stage, distinguished over the scenarios s).

 $A d \leq b$: especially scheduling constraints for outages

 $B \ p_s \leq b_s$: production/ fuel level constraints (can be included in $T \ d + W \ p_s \leq h_s$) Main constraints $T \ d + W \ p_s \leq h_s$, coupling maintenance decisions to null T2 production:

$$\forall i, k, s, w, \quad p_{i,k,s,w} \leqslant \overline{\mathsf{P}}_{i,w}(d_{i,k,w-\mathsf{D}\mathsf{a}_{i,k}} - d_{i,k+1,w}) \tag{1}$$

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Solving limits

- Strong limiting factor: the sizes of instances
- LP relaxation: not computable in 1h for difficult instances (B8-B9) even with deterministic problem and aggregated production time steps to weeks with Cplex 12.3. OK with 12.5
- With restricted time windows, good B& B convergence
- B&B search efficient restricting the size (1 stochastic scenario, time steps aggregated to weeks, 3 cycles and 120 weeks max with time windows)
- Dual heuristics compute high quality dual bounds for the whole problem.
- Efficient matheuristics to tackle the size of the full problem.

 \implies In this talk, for the robust extension, we consider (reduced) instances where straightforward B&B solving is efficient.

Dupin, N., Talbi, E.: Dual Heuristics and New Lower Bounds for the Challenge EURO/ROADEF 2010. Matheuristics 2016 pp. 60–71 (2016) Dupin, N., Talbi, E.: Dual heuristics and new dual bounds to schedule the maintenances of nuclear power plants, submitted, preprint available in arXiv.

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- 3 Lighter robustness approach
- 4 Conclusions and perspectives

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Robust problem

- Outage durations are uncertain due to possible delays of the maintenance operations.
- Impact on the planning with linking constraints on outages, production capacities . . .
- We consider here only the outage prolongation as uncertain.
- We want to face off the worst scenario of prolongation in the whole planning considering outages prolongation in a given uncertainty set.
- \implies Operational/industrial requirements meet robust optimization

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Uncertainty sets

- Each prolongation $\delta_{i,k}$ has a maximal duration $\overline{\delta_{i,k}}$.
- Interval uncertainty: $\Omega_{worst} = \prod_{i,k} \llbracket 0, \overline{\delta_{i,k}} \rrbracket$.
- Maximal budget maximal of extreme delays: $\Omega_{budget}^{\Gamma} = \left\{ \delta \in \Omega_{worst} \middle| \sum_{i,k} \delta_{i,k} \leqslant \Gamma \right\}.$

• Cardinality restriction:

$$\Omega_{card}^{N} = \left\{ \delta \middle| \exists (\varepsilon_{i,k}) \in \{0,1\}, \forall i, k, \delta_{i,k} \leqslant \overline{\delta_{i,k}} \varepsilon_{i,k} \text{ and } \sum_{i,k} \varepsilon_{i,k} \leqslant N \right\}$$

 \Longrightarrow Discrete uncertainty sets, Ω in the following, discretizing all the possible scenarios.

 \implies main case: N = 1 (or $\Gamma = 1$) and $\overline{\delta_{i,k}} = 1$ (and will make enough difficulties . . .)

Robustness definition

Robustness: We want to face off the worst case in the uncertainty set $\Omega,$ minimizing the worst expected cost.

Second level decisions can be adjusted after the uncertainty outcomes. It leads to a Min-Max-Min scheme:

$$\min_d c_D.d + \mathcal{Q}(d)$$
 with:
 $A.d \leqslant b$

$$\begin{array}{lll} \mathcal{Q}(d) = & \max_{\delta \in \Omega} \mathcal{Q}(d, \delta) & \text{and} & \mathcal{Q}(d, \delta) = & \min_{x} & c_{p}.p \\ & & T(\delta).d + W.p \leqslant h(\delta) \end{array}$$

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Uncertainty set, why not a continuous uncertainty set?

- Nicer with Bertsimas and Sim model, to use duality and usual 2-stage robust framework: $\Omega_{budget}^{\Gamma} = \left\{ \delta \in \prod_{i,k} [0, \overline{\delta_{i,k}}] \middle| \sum_{i,k} \delta_{i,k} \leqslant \Gamma \right\}.$
- Uncertainty in constraints, implying feasibility issues (it would have been better with cost uncertainty)
- WORSE: Uncertainty is non linear in constraints (and absolutely discrete in the w' indexes of d_{i,k,w'}):

$$\forall i, k, \delta \in \Omega, w, \quad p_{i,k,\delta,w} \leqslant \overline{\mathbf{P}}_{i,w}(d_{i,k,w-\mathbf{Da}_{i,k}-\delta_{i,k}}-d_{i,k+1,w})$$
(2)

 \implies No alternative to consider a discrete set of scenarios

Linearization having discrete scenarios

Linearization with discrete scenarios $\delta \in \Omega$, MIP to be solved with Benders decomposition:

$$egin{aligned} \min_{d\in\{0,1\}^n imes\mathbb{R}^m_+,p_\delta\geqslant 0} & cd+C^{rob} \ & A\ d & \leqslant\ b \ & orall \delta & T_\delta\ x+W\ p_\delta & \leqslant\ h \ & orall \delta & q\ p_\delta & \leqslant\ C^{rob} \end{aligned}$$

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Benders decomposition for our MIP

 Variable partitioning: first level variables 'z': d_{i,k,w}, r_{i,k} and C^{robust}. Other variables 'y' depend on the scenarios.

$$\min_{z,y \ge 0} cz \tag{3}$$

$$Az \geqslant a$$
 (4)

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$$Tz + Wy \ge b \tag{5}$$

 Master Problem min cz s.t Az ≥ a and cuts generated by the subproblems. (projection of the constraints Tz + Wy ≥ b in the space of z variables).

Generating Benders cuts

 Here just feasibility cuts: z⁰ given, is it possible for all the prolongation scenarios to have a production planning with cost at most C^{robust}? Transforming into optimization problem and using duality:

$$\min \eta \qquad \max(b - T.z^0).v > 0? \qquad (6)$$

$$Wy + \eta \ge b - T.z^0 \qquad = \qquad W^T.v \le 0 \tag{7}$$

$$\sum_{i} v_i \leqslant 1 \tag{8}$$

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$$\eta, y \ge 0 \qquad \qquad v \ge 0 \tag{9}$$

• Benders Reformulation: for all extreme ray v of W^T , we have cuts $(d - T.z).v \leq 0.$

Variable definition

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- Continuous variables:
 - $r_{i,k}$: refueling quantities for the refueling in outage (i, k).
 - *p*_{i,k,s,δ,t} nuclear (T2) productions T1.
 - $p_{j,s,\delta,t}$ non nuclear (T1) productions levels.
 - Residual stocks $x_{i,s,\delta}^{fin}$ (for cost function, to avoid end-of-side effects)
- Dependent variables: fuel stocks $x_{i,k,s,\delta}^{init}$, $x_{i,k,s,\delta}^{fin}$.

 $\forall \delta$,

min
$$\sum_{i,k} \mathbf{C}_{i,k}^{rld} r_{i,k} + \sum_{i,k,w} \mathbf{C}_{i,k,w}^{pen} (d_{i,k,w} - d_{i,k,w-1}) + C^{robust}$$
 (10)

$$\forall i, k, w, \qquad \qquad d_{i,k,w-1} \leqslant d_{i,k,w} \tag{11}$$

$$\forall i, k, \qquad \qquad \mathsf{Rmin}_{i,k} \ d_{i,k,W} \leqslant r_{i,k} \leqslant \mathsf{Rmax}_{i,k} \ d_{i,k,W} \qquad (12)$$

$$\sum_{j,s,t} \pi_s \mathbf{C}_{j,s,t}^{prd} \mathbf{D}^t \ p_{j,s,\delta,t} - \sum_{i,s} \pi_s \mathbf{C}_i^{val} x_{i,s,\delta,T} \leqslant C^{robust}$$
(13)

$$\forall i, s, \delta, \qquad \qquad x_{i,-1,s}^{init} = \mathbf{X}\mathbf{i}_i \tag{14}$$

$$\forall j, t, s, \delta, \qquad \qquad \mathsf{Pmin}_{j,t}^{s} \leqslant p_{j,s,\delta,t} \leqslant \mathsf{Pmax}_{j,t}^{s} \tag{15}$$

$$\forall i, t, s, \delta, \qquad \qquad 0 \leqslant p_{i,k,s,\delta,t} \leqslant \mathbf{Pmax}_i^t(d_{i,k,w_t} - \mathbf{Da}_{i,k,\delta} - d_{i,k+1,w_t}) \tag{16}$$

$$\forall i, t, s, \delta, m, \qquad \qquad \frac{P_{i,k,s,\delta,t}}{\mathsf{Pmax}_i^t} \leqslant \frac{\mathsf{c}_{i,k,m-1}^{-} - \mathsf{c}_{i,k,m}}{\mathsf{f}_{i,k,m-1} - \mathsf{f}_{i,k,m}} (\mathsf{x}_{i,s,\delta,t} - \mathsf{f}_{i,k,m}) + \mathsf{c}_{i,k,m} \tag{17}$$

$$\forall s, t, \delta, \qquad \sum_{i,k} p_{i,k,s,\delta,t} + \sum_{j} p_{j,s,\delta,t} = \mathsf{Dem}^{t,s}$$
(18)

$$\forall i, k, s, \delta, \qquad \qquad 0 \leqslant x_{i,k,s,\delta}^{init} \leqslant \mathbf{Smax}_{i,k} \tag{19}$$

$$\forall i, t, s, \delta, \qquad \qquad x_{i,k,s,\delta}^{fin} = x_{i,k,s,\delta}^{init} - \sum_{t} \mathbf{D}^{t} p_{i,k,s,\delta,t}$$
(20)

$$\forall i, k, s, \delta, \qquad \qquad x_{i,k,s,\delta}^{fin} \leqslant \operatorname{Amax}_{i,k+1} + (\operatorname{Smax}_{i,k} - \operatorname{Amax}_{i,k+1}) d_{i,k+1,W} \tag{21}$$

$$\forall i, t, s, \delta, \qquad \qquad x_{i,k,s,\delta}^{init} - \mathbf{Bo}_{i,k} = r_{i,k} + \frac{\mathbf{Q}_{i,k-1}}{\mathbf{Q}_{i,k}} (x_{i,k-1,s,\delta}^{fin} - \mathbf{Bo}_{i,k-1})$$
(22)

$$\forall i, k, s, \delta, t, \quad x_{i,s,\delta,t} \leqslant x_{i,k,s,\delta}^{init} - \sum_{t' \leqslant t} \mathbf{D}^{t'} \, p_{i,k,s,\delta,t'} + M_i \left(1 - d_{i,k,w_t} + d_{i,k-1,w_t}\right) \tag{23}$$

$$\forall w, \delta, \qquad \sum_{i,k} \operatorname{Pmax}_{i}^{w}(d_{i,k,w} - d_{i,k,w} - \mathbf{D}_{\mathbf{a}_{i,k,\delta}}) \leqslant \operatorname{Imax}$$
(24)

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$$\sum_{i,k,w} \alpha_{i,k,w}^{(1)} \mathsf{Pmax}_{i}^{w}(d_{i,k,w-\mathsf{Da}_{i,k,\delta}} - d_{i,k+1,w}) + \sum_{i,k} \alpha_{i,k}^{(4)} \left(\mathsf{q}_{0,k-1} \mathsf{Xi}_{i} + \sum_{l=0}^{k-1} \mathsf{q}_{l+1,k-1}(r_{i,l+1} - \mathsf{Bo}_{i,l}) \right)$$

$$+\sum_{i,k} \alpha_{i,k}^{(2)} \left(\mathsf{Smax}_{i,k} \left(1 + d_{i,k+1,W} - d_{i,k,W} \right) + \mathfrak{q}_{0,k-1} \mathsf{Xi}_i + \sum_{l=0}^{k-1} \mathfrak{q}_{l+1,k-1} (r_{i,l+1} - \mathsf{Bo}_{i,l}) \right)$$

$$+\sum_{i,k} \alpha_{i,k}^{(3)} \left(\mathsf{Smax}_{i,k} - (\mathsf{Smax}_{i,k} - \mathsf{Amax}_{i,k+1}) \ d_{i,k+1,W} - \mathsf{q}_{0,k-1} \mathsf{Xi}_{i} - \sum_{l=0}^{k-1} \mathsf{q}_{l+1,k-1}(r_{i,l+1} - \mathsf{Bo}_{i,l}) \right)$$

$$+ \sum_{i,k} \alpha_{i,k}^{(5)} \left(\mathsf{Smax}_{i,k} - \mathsf{q}_{0,k-1} \mathsf{Xi}_{i} - \sum_{l=0}^{k-1} \mathsf{q}_{l+1,k-1} (r_{i,l+1} - \mathsf{Bo}_{i,l}) \right)$$

$$+ \sum_{w} \alpha_{w}^{(17)} \left(1 - \sum_{(i,k) \in \mathsf{A17}} (d_{i,k,w} - \mathsf{Da}_{i,k,\delta} - d_{i,k,w} - \mathsf{Da}_{i,k,\delta} - \mathsf{Se17}) \right)$$

$$+\sum_{\mathbf{w}} \alpha_{\mathbf{w}}^{(18)} \left(1 - \sum_{(i,k) \in \mathbf{A18}} (d_{i,k,\mathbf{w}} - d_{i,k,\mathbf{w}-\mathbf{Se18}}) + (d_{i,k,\mathbf{w}-\mathbf{Da}_{i,k,\delta}} - d_{i,k,\mathbf{w}-\mathbf{Da}_{i,k,\delta}-\mathbf{Se18}}) \right)$$

$$+\sum_{w} \alpha_{w}^{(20)} \left(\mathbf{N}_{w}^{20} - \sum_{(i,k) \in \mathbf{A}_{w}^{20}} (d_{i,k,w} - d_{i,k,w-\mathbf{Da}_{i,k,\delta}}) \right) + \sum_{w} \alpha_{w}^{(21)} \left(\mathbf{I}^{max} - \sum_{(i,k)} \mathbf{Pmax}_{i}^{w} (d_{i,k,w} - d_{i,k,w-\mathbf{Da}_{i,k,\delta}}) \right)$$

$$-\sum_{w}\beta_{w}^{(1)}\mathsf{P}_{j,w}^{min} + \sum_{w}\beta_{w}^{(2)}\mathsf{P}_{j,w}^{max} + \sum_{w}\beta_{w}^{(3)}\mathsf{Dem}^{w} + \sum_{w}\beta_{w}^{(4)}(\mathsf{P}_{0,w}^{max} - \mathsf{Dem}^{w}) + \gamma C^{rob} \ge 0$$

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Implementation points

Challenging difficulty: very large scale problem.

- Numerical instability due to propagation of rounding errors in Benders cuts.
- Too conservative approach: scheduling constraint induce tight planning in summers, there is frequently no 100% robust solution
- \Longrightarrow Needs for another definition of robustness, for modeling and solving issues

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Specific case of constraints CT14 and CT15

Former robust constraints for CT14 and CT15:

$$\forall \delta, w, \quad \sum_{(i,k) \in \mathbf{A14}} (d_{i,k,w} - d_{i,k,w-(\mathbf{Da}_{i,k,\delta} + \mathbf{Se14})^+}) \leqslant 1$$
 (25)

$$\forall \delta, w \in [\mathbf{d}_{15}, \mathbf{f}_{15}], \quad \sum_{(i,k) \in \mathbf{A15}} (d_{i,k,w} - d_{i,k,w-(\mathbf{Da}_{i,k,\delta} + \mathbf{Se15})^+}) \leqslant 1$$
(26)

The robustness of CT14 and CT15 is equivalent to Soyster's approach Robust CT14 and CT15 constraints are equivalent to the deterministic constraints with $\mathbf{Da}_{i,k} = \overline{\mathbf{Da}}_{i,k}$, using Soyster's results:

$$\forall w, \quad \sum_{(i,k)\in \mathbf{A14}} (d_{i,k,w} - d_{i,k,w-(\overline{\mathbf{Da}}_{i,k}+\mathbf{Se14})^+}) \leqslant 1 \tag{27}$$

$$\forall w \in [\mathbf{d}_{15}, \mathbf{f}_{15}], \quad \sum_{(i,k) \in \mathbf{A15}} (d_{i,k,w} - d_{i,k,w - (\overline{\mathbf{Da}}_{i,k} + \mathbf{Se15})^+}) \leqslant 1$$
(28)

 \implies Trick can be used to reduce the size of previous model \implies As CT14 and CT15 predominant to design good robust solutions, robust constraints with deterministic solution still infeasible in most of the instances.

Robustified approach

For all constraints $c \in CT14$ and $c \in CT15$, continuous variables $z_{c,w}^{(14)}, z_{c,w}^{(15)} \ge 0$ are introduced to penalize robust violations, paying cost **Cpen**^{rob} for violations.

We try to minimize $f_{obj}^{rob} = \sum_{w} \mathbf{Cpen}^{rob}(z_{c,w}^{(14)} + z_{c,w}^{(15)})$ and also with f_{obj}^{det} the previous objective

We add to the previous deterministic formulation the constraints:

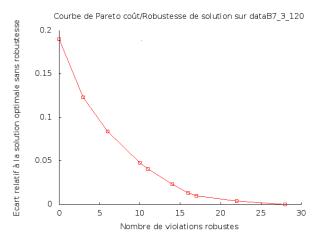
$$\begin{aligned} \forall w, c \in CT14 \quad \sum_{(i,k)\in \mathbf{A14}^c} (d_{i,k,w} - d_{i,k,w-(\overline{\mathbf{Da}}_{i,k}+\mathbf{Se14})^+}) \leqslant 1 + z_{c,w}^{(14)} \\ \forall c \in CT15, w \in [\mathbf{d}_{15}^c, \mathbf{f}_{15}^c], \quad \sum_{(i,k)\in \mathbf{A15}^c} (d_{i,k,w} - d_{i,k,w-(\overline{\mathbf{Da}}_{i,k}+\mathbf{Se15})^+}) \leqslant 1 + z_{c,w}^{(15)} \end{aligned}$$

Solving facts

- Weighted-sum robustified MIP problem with a similar size than the deterministic one.
- Same set of feasible solutions than the deterministic MIP: furnish robustified solutions for all instances
- Similar B& B characteristics than the deterministic MIP.
- Robust trade-off can help MIP solvers to cut off solutions. Difficulties in deterministic MIP that lots of solutions have similar cost, known bottleneck for B&B.
- Computation of Pareto fronts of the best compromise solutions to trade off cost/robustness.
- *f*^{rob}_{obj} has discrete values, a good point for a ε-constraint method for bi-objective optimization. Also dichotomic search (first phase of TPM method) applies in this bi-objective case.

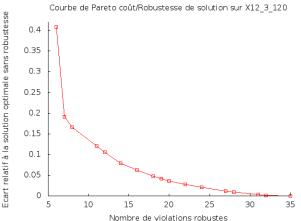
 $\implies \text{Simple approach, but efficient in a operational standpoint and using a simple and efficient methodology.}$

Courbe de Pareto Cout/Robustesse



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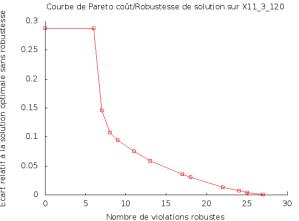
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2-stage Robust model





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Conclusions and perspectives

Conclusions:

- Robust optimization with an hypothesis significantly different from the state of the art: discrete uncertainty.
- Benders approach: numerical difficulties.
- Simple robustified approach: efficient, based on deterministic resolution, consistent to give feasible solutions.
- Robustness is a trade-off to cut off non robust solutions as lots of solutions have similar cost.

Perspectives:

- Stability objective through the dynamic reoptimizations.
- Matheuristic construction of Pareto Fronts for the real-size instances