Can we approximate a weight set decomposition?

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These results have been acquired during the

vOpt Project

Exact Efficient Solution of Mixed-Integer Programming Problems with Multiple Objective Functions

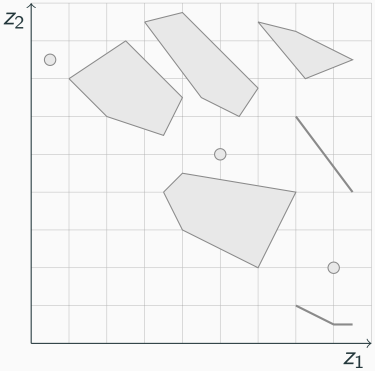
https://vopt-anr-dfg.univ-nantes.fr/

DFG grant RU 1524/4-1

I. Introduction

Nondominance and Weighted Sum

$min_{x \in X}\ z(x) = Cx$ with $Y = z(X)$.



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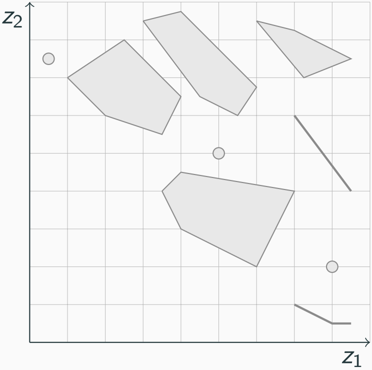

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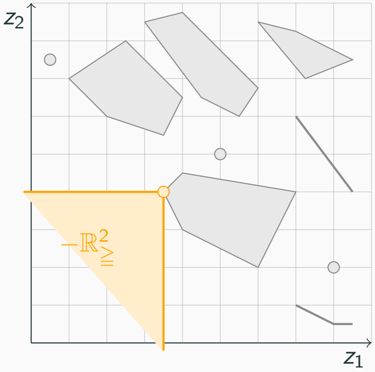

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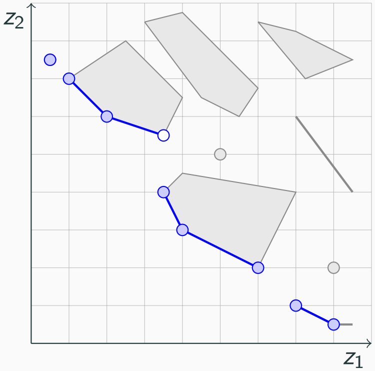

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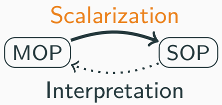

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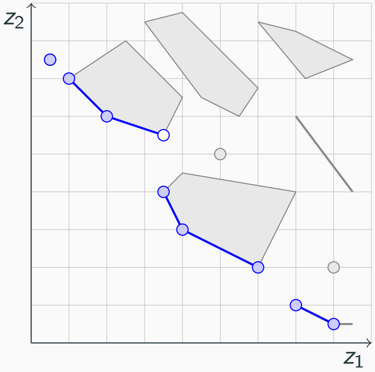

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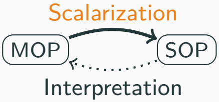

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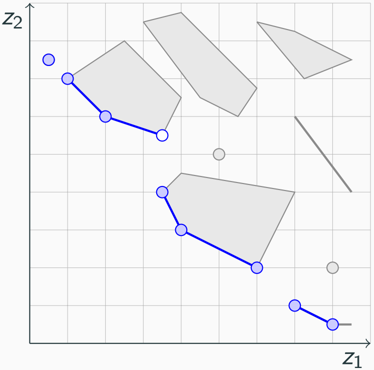

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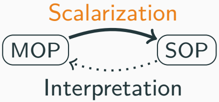

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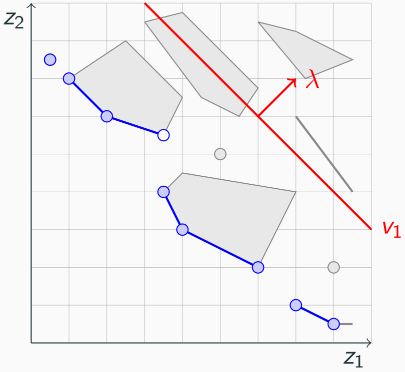

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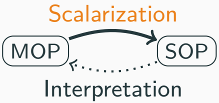

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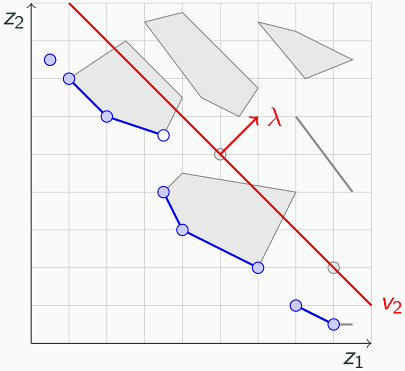

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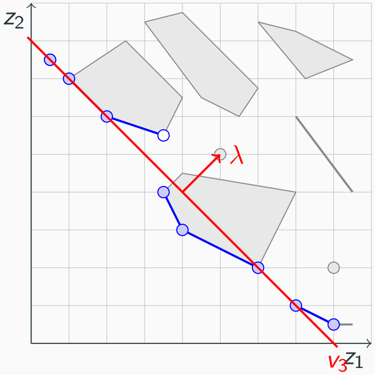

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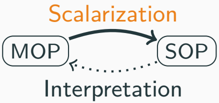

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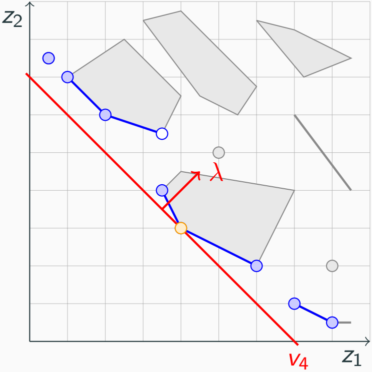

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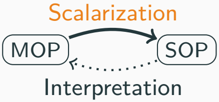

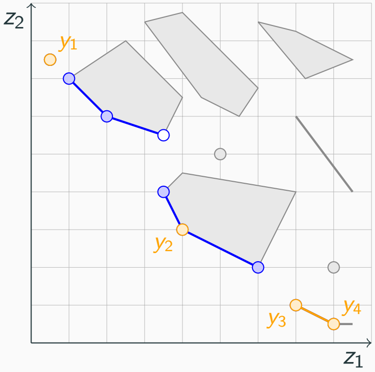

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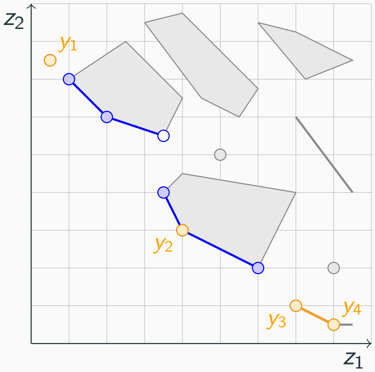

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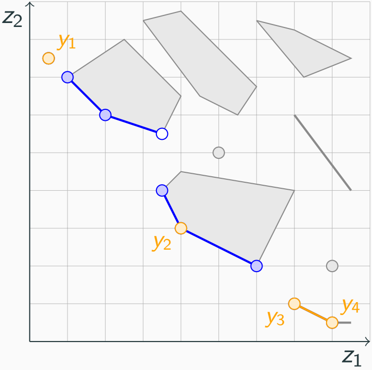

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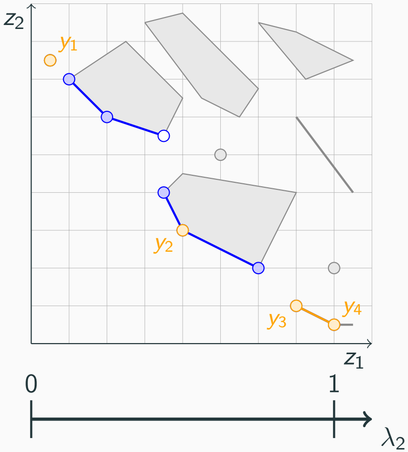

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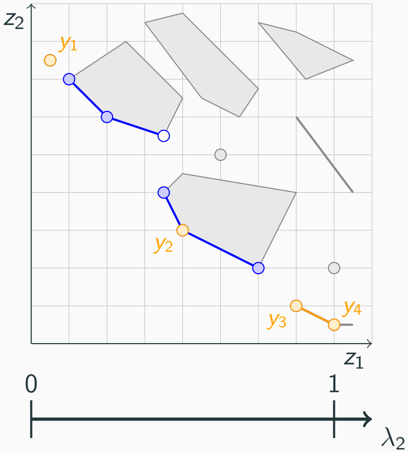

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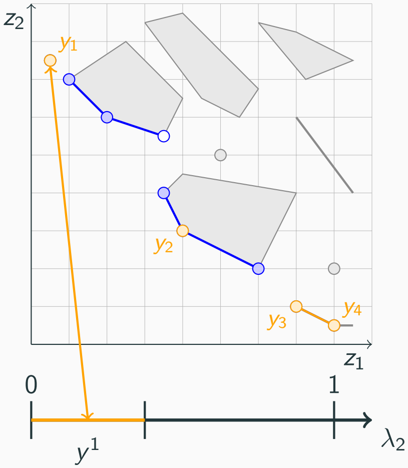

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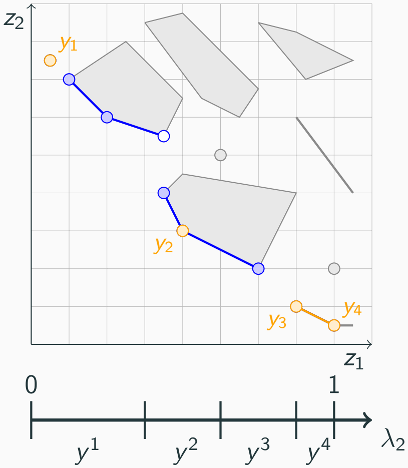

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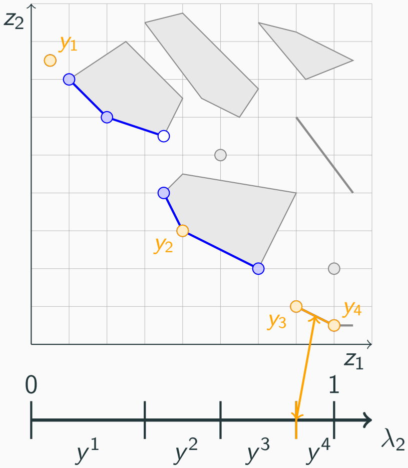
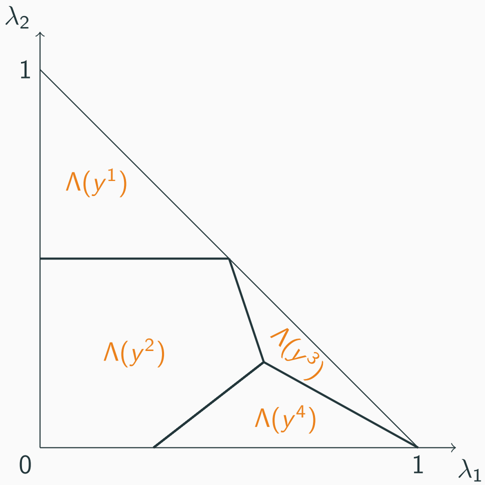


Weight Set for Tri Objective Problems



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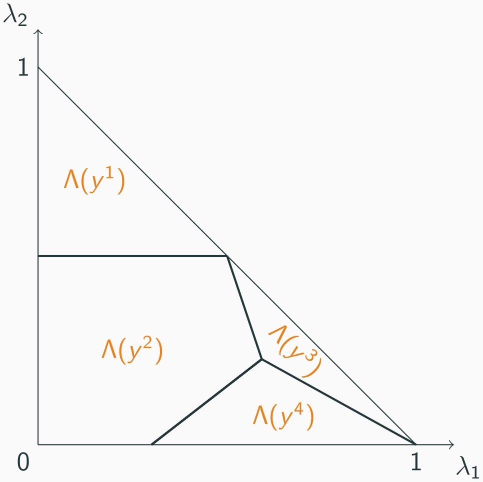
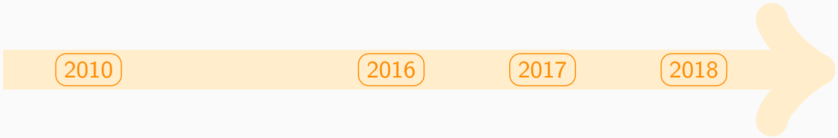


Weight set components are convex polytopes and the components of the ESN images decompose the weight set.

Recent Work



Recent Work

2010

Przybylski et al. First exact algorithm for three objectives, with recursion usable for > 3 objectives.

Özpeynirci and Köksalan First non-recursive exact algorithm for more than three objectives.

2016

Alves and Costa Exact algorithm for three objectives.

2017

Bökler and Mutzel First output sensitive exact algorithm for finding ESN images for combinatorial problems.

2018

Halffmann et al. Output sensitive graphical exact algorithm for the weight set decomposition using Dichotomic Search (in progress).

II. Approximation of a Weight Set Decomposition

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What about approximation algorithms or heuristics? This has not been done before!

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- An approximation factor or bound and additional requirements.

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- Easy to comprehend and visualize.
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Hausdorff Distance

- $\delta(L(y), \Lambda(y)) := \max\{\max\{D(\lambda, L(y)), \lambda \in \Lambda\} \max\{D(l, \Lambda(y)), l \in L(y)\}\}$,
  with $D(f, G) := \min\{d(f, g), g \in G\}$ and $d(.,.)$ is the euclidean metric.
- Use as measure $1 - \frac{\delta(L(y),\Lambda(y))}{\max_{a,b\in\Lambda(y)} d(a,b)}$.
- Easier to compute but harder to comprehend.

Approximation of a Weight Set Component

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Instead of $L(y)$ and $\Lambda(y)$, use $\underline{L}(y)$ and $\overline{L}(y)$, simplifies the computation of the quality measure.

Basic Definitions

Weight Set Decomposition Approximation Algorithm

Given an instance of a multiobjective problem, an α-Weight Set Decomposition Approximation Algorithm with $0 < \alpha \leq 1$ returns for a subset $\emptyset \neq S \subseteq Y_{ESN}$ of the extreme supported nondominated images sets $L(y) \subseteq \Lambda, y \in S$ such that each $L(y)$ approximates $\Lambda(y)$ by at least α. Further, the algorithm is a polynomially α-Weight Set Decomposition Approximation Algorithm if it additionally runs in polynomial time.

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Weight Set Decomposition Approximation Heuristic

Each iteration i the algorithm returns a subset S^i and an α^i-approximation of the weight set component of each $y \in S^i$. We require that $\lim_{i\to\infty} S^i = Y_{ESN}$ and $\lim_{i\to\infty} \alpha^i = 1$ and each iteration can be done in polynomial time.

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Convergence Rate

Let $a^i(y)$ the approximation factor for $\Lambda(y)$ in iteration i. We measure the convergence rate for one component of the heuristic by

$$\limsup_{i\to\infty} \frac{a^{i+1}(y)}{a^i(y)}.$$

III. Approximation Algorithms and Heuristics

Overview of the Methods

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Subdivide the weight set into easy computable full-dimensional polytopes.
Evaluate the vertices:
If one solution is optimal for all vertices, then this polytope belongs to the component of this solution, otherwise subdivide again.

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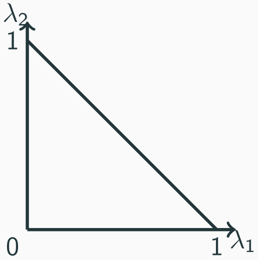

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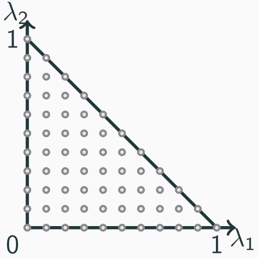

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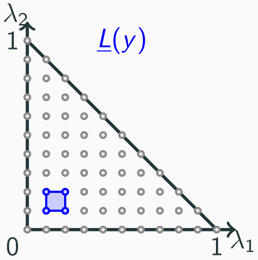

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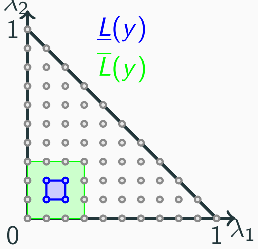

Overview of the Methods

Point Search

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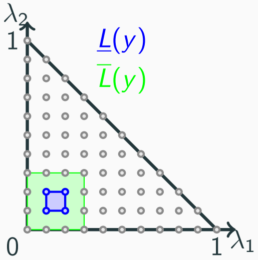

Line Search

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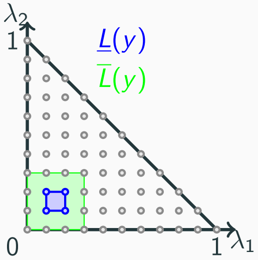

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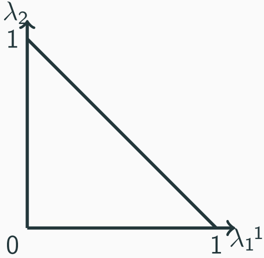

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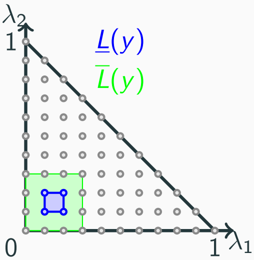

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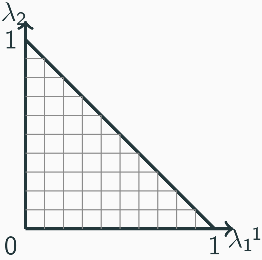

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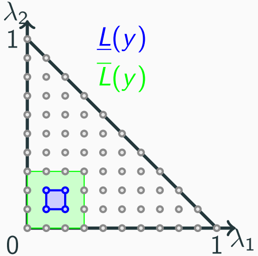

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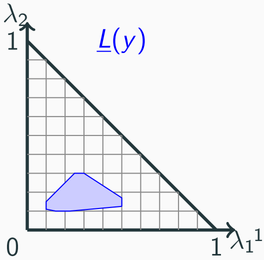

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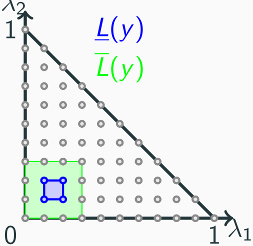

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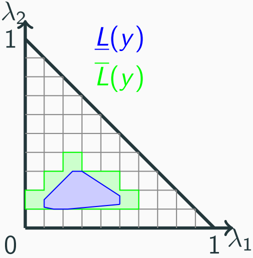
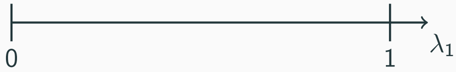


Point Search: The Biobjective Case



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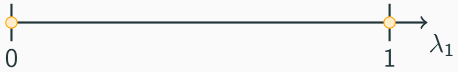

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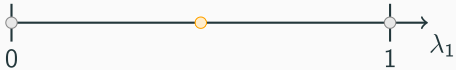

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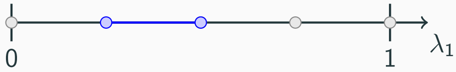

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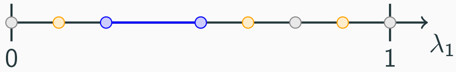

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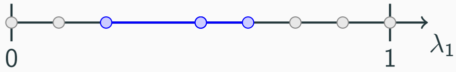

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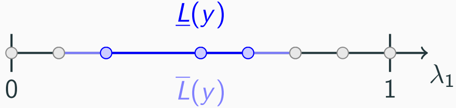

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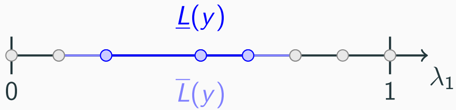

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Given $\epsilon > 0$ the Point Search gives an $\frac{1}{3}$-approximation (Hypervolume) of all components that are found in $\mathcal{O}(\frac{1}{\epsilon} T_{WS})$. In order to definitely return at least one weight set component, we require $\frac{1}{\epsilon} > |Y_{ESN}|$. This method has a convergence rate of $\frac{1}{2}$.

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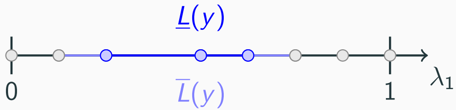

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Due to the choice of ϵ this can be only used as an heuristic.

Point Search: The Multiobjective Case

Different Subdivisions:

Point Search: The Multiobjective Case

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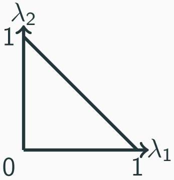

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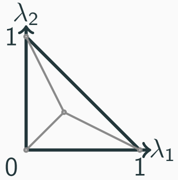

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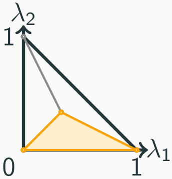

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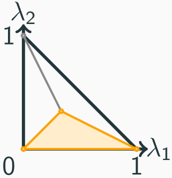

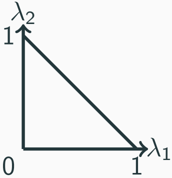

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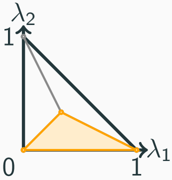

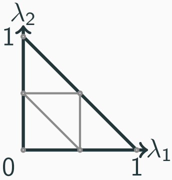

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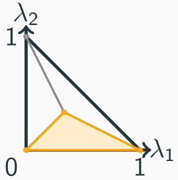

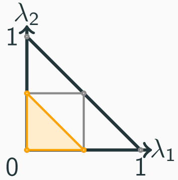

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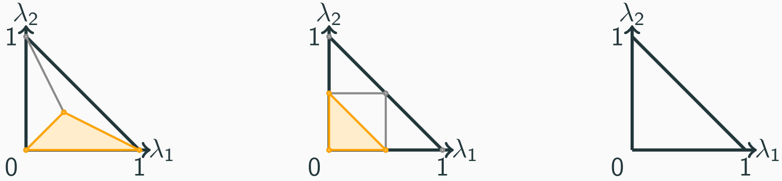

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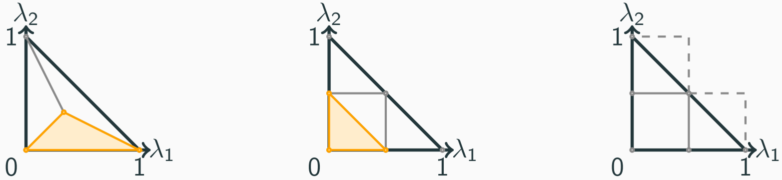

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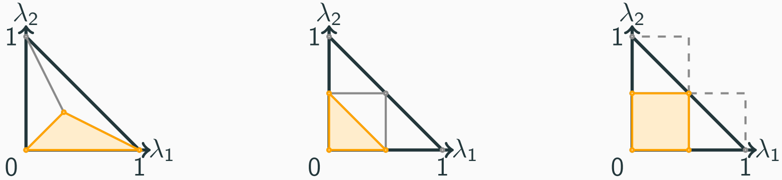

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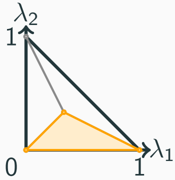

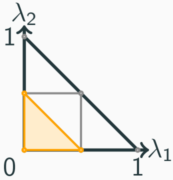

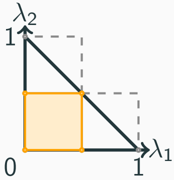

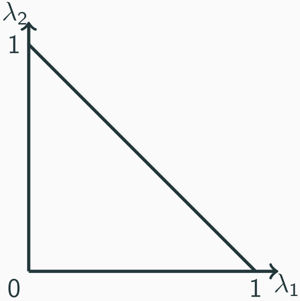

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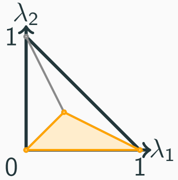

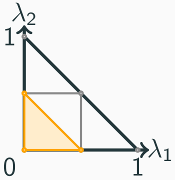

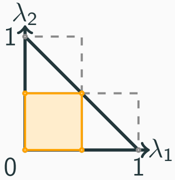

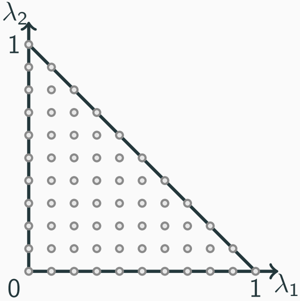

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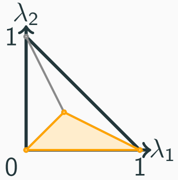

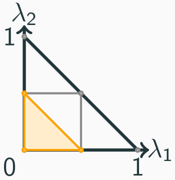

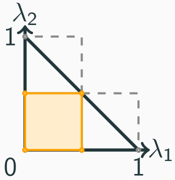

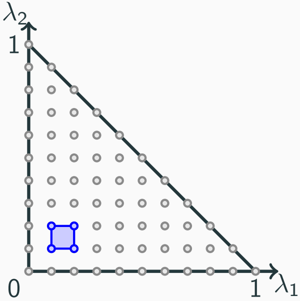

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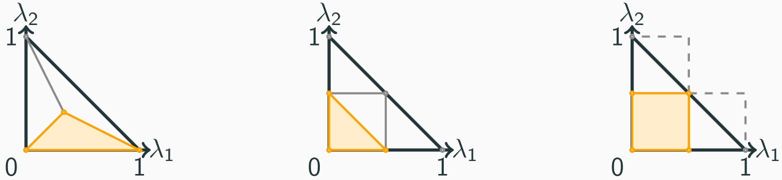

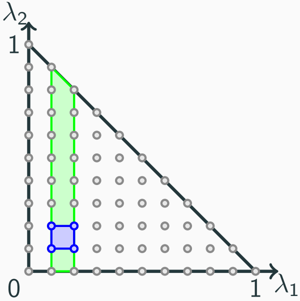

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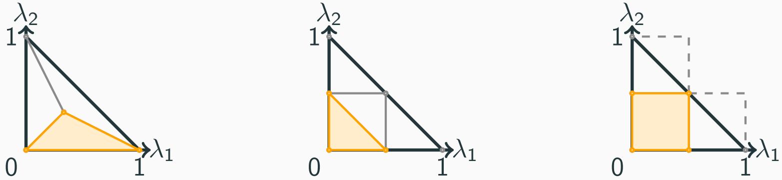

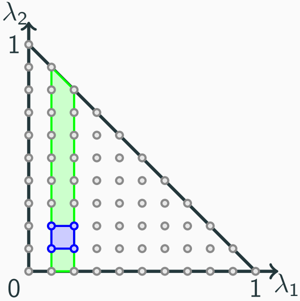

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Given $\epsilon > 0$ the Point Search gives an ϵ-approximation (Hausdorff) of all components that are found in $\mathcal{O}(\frac{1}{\epsilon} T_{WS})$.

Point Search: The Multiobjective Case

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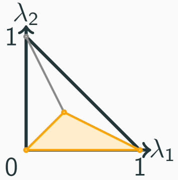

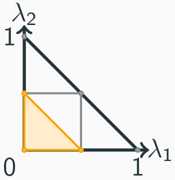

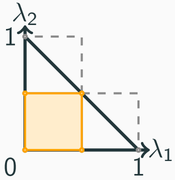

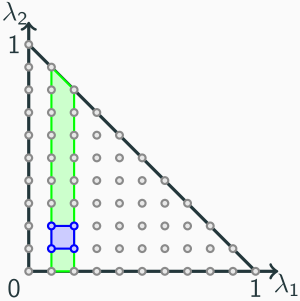

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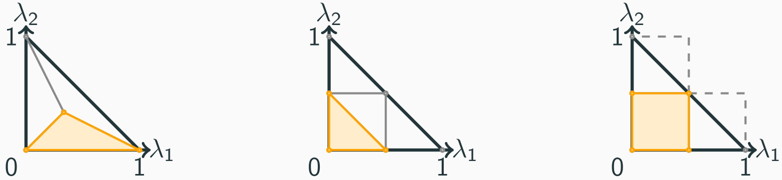

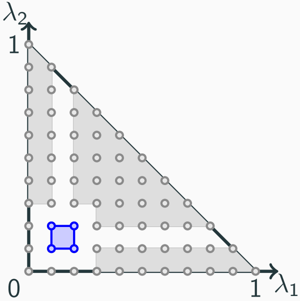

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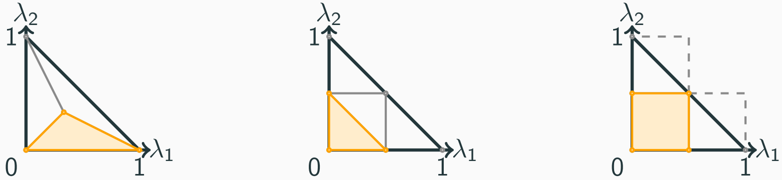

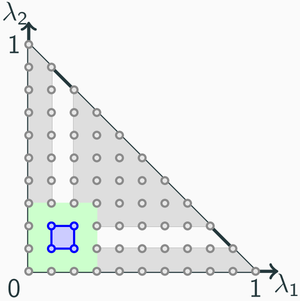

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Point Search: Example

Assignment Problem with 10 items:

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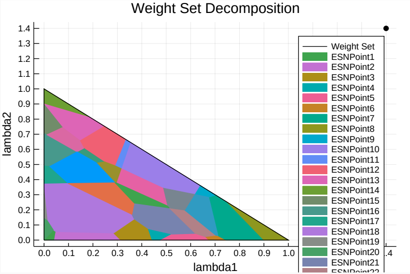

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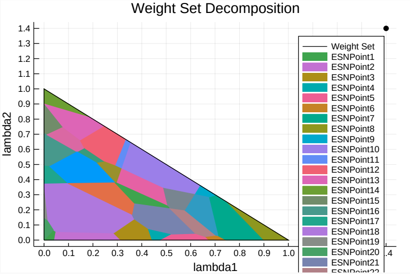

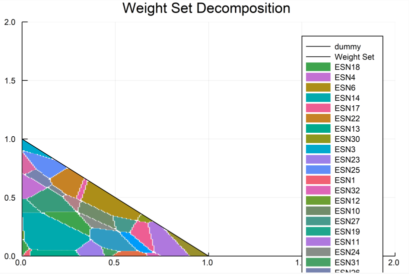

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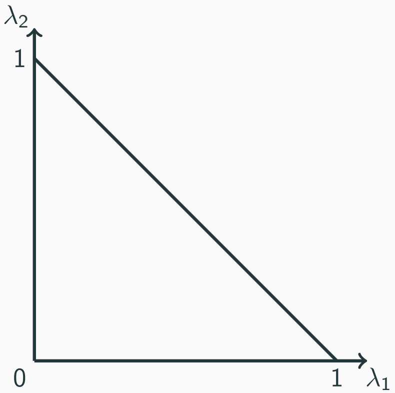

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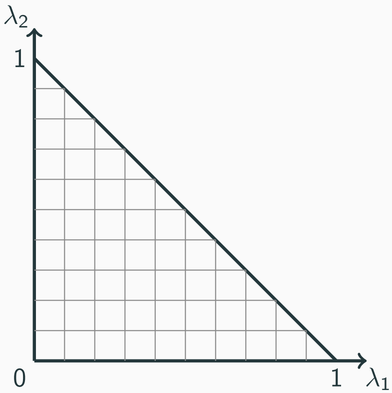

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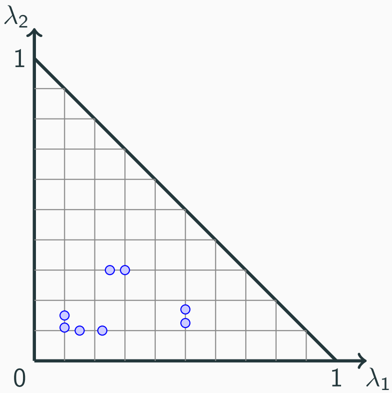

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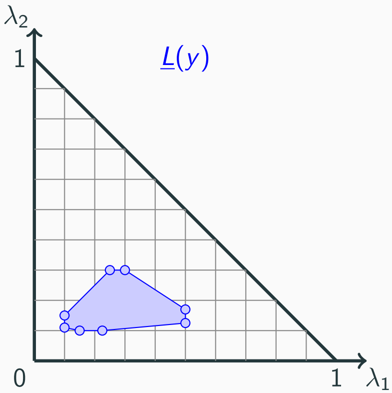

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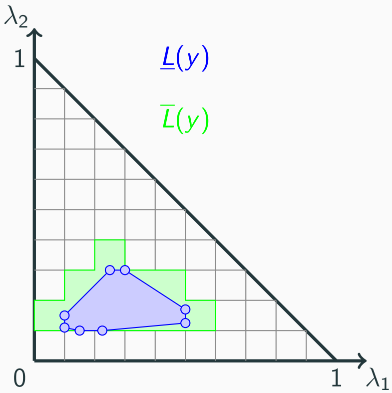

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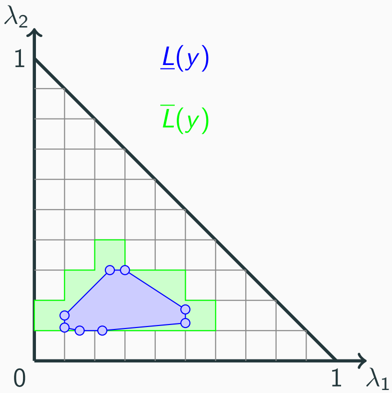

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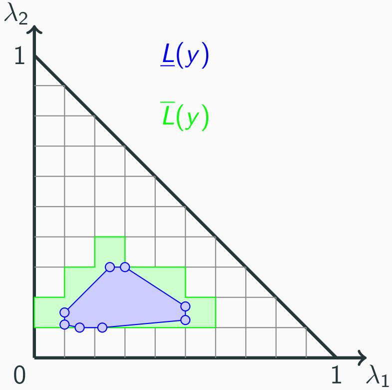
Given $\epsilon > 0$ the Line Search gives an $1 - \epsilon$-approximation (Hausdorff) of all components that are found in $\mathcal{O}(\frac{1}{\epsilon}|Y_{ESN}|T_{WS})$.

Line Search



Result

Given $\epsilon > 0$ the Line Search gives an $1-\epsilon$-approximation (Hausdorff) of all components that are found in $\mathcal{O}(\frac{1}{\epsilon}|Y_{ESN}|T_{WS})$.

Not polynomial, hard to find an ϵ such that at least one component is found. Same problem as for the Point Search for $\dim \Lambda \geq 2$.

IV. Open Question and Outlook

Open Question

Approximate Weight Set Component Problem

Let $\alpha > 0$. Given an instance of a multiobjective problem where the weighted sum problem is solvable in polynomial time. Can we, in terms of our definition, approximate at least one weight set component by factor α in polynomial time?

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Let $\alpha > 0$. Given an instance of a multiobjective problem where the weighted sum problem is solvable in polynomial time. Can we, in terms of our definition, approximate at least one weight set component by factor α in polynomial time?

Conjecture: This problem is *APX-hard*, however proving this is hard by itself, as it is a mixture of an optimization and a search problem.

Outlook

- Progress on the approximation quality and convergence rate.

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- Progress on the approximation quality and convergence rate.
- Answer the general approximability of the weight set decomposition.

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- Progress on the approximation quality and convergence rate.
- Answer the general approximability of the weight set decomposition.
- What happens, if we use approximation algorithms to solve the weighted sum problem?

Thank you for your attention!

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