On two speeding-up techniques for the computation of multi-objective shortest paths with a label setting algorithm

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with
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vOpt Research Project

Exact Efficient Solution of Mixed Integer Programming Problems with Multiple Objective Functions (ANR/DFG-14-CE35-0034-01)

1. Introduction

Assumptions and notations

- On the problem:
  - $G(N, A, C)$, a static, connected, directed and valued graph
  - $p - \sum$, i.e. p linear objectives with $p \geqslant 2$
  - Compute 1-to-all (source s to other nodes) shortest paths
- On the instance:
  - No specific topology for G
  - p positive costs on arcs
- On the solutions:
  - No preference on shortest paths
  - $s \in N$ given:
    efficient paths over p objectives from s to all $t \in N \backslash \{s\}$,
    X_E, a complete set of efficient paths,
    $Y_N = Z(X_E)$, the set of non-dominated points
- On the algorithm:
  - Label setting principle
  - Martins' algorithm (1984)

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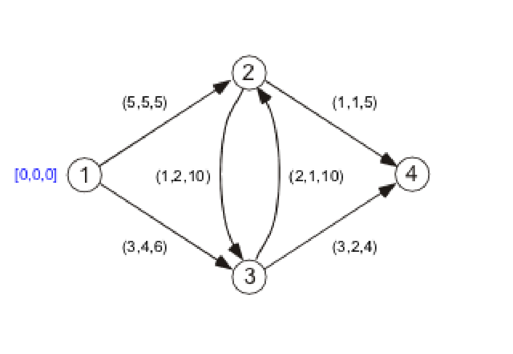
Ingredients and principle of Martins' algorithm

- Temporary and permanent labels
- Lexicographic selection of a temporary label
- Propagation principle over outgoing arcs
- All permanent labels correspond to efficient paths
- Pruning temporary labels on nodes by dominance



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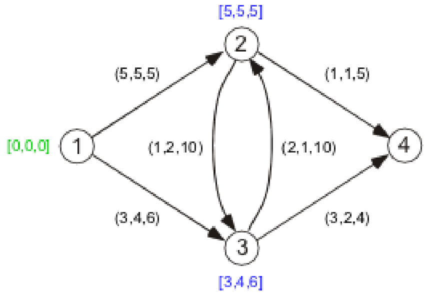

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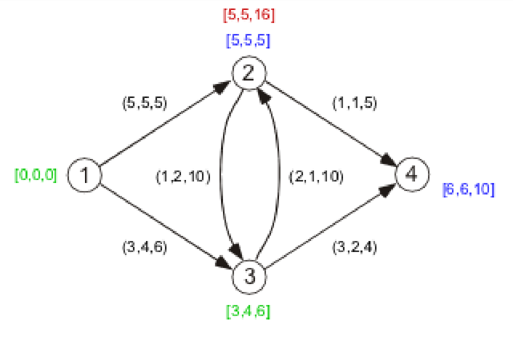

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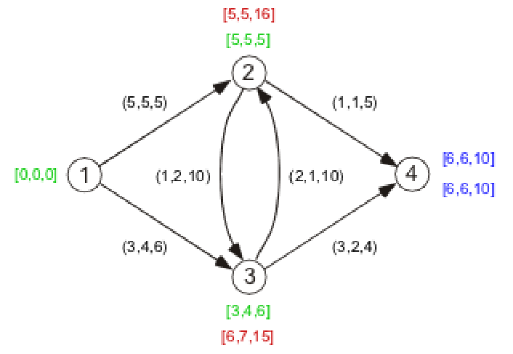

Motivation and Questions

- Motivations: good base
    - to investigate the influence of an instance on the algorithm
    - to measure the impact of additional components on the algorithm
    - to develop an implementation to be integrated into vOptSolver

- Questions:
    - Concerning the maintenance of non-dominated temporary labels (operations of comparison, insertion, deletion) on nodes:

      What is the added value of an advanced data structure for maintaining on nodes during the iterations of the algorithm?

    - Concerning the generation of temporary labels on nodes:

      What is the added value of a two-directional strategy on the total number of temporary labels generated by the algorithm?

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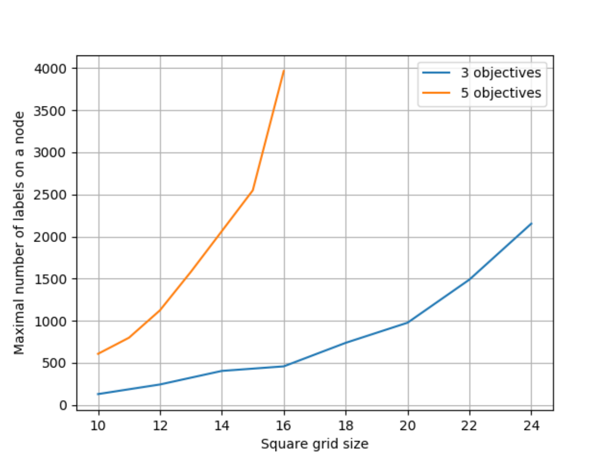
2. On the strategy to maintain temporary labels

Observation

- Maximal number of labels (temporary and permanents) on 1 node:



Situation

- Maintaining non-dominated labels on nodes requires operations of comparison, insertion, deletion.
- A multi-dimensional data structure is then required for storing the information and facilitating these operations during the iterations of the algorithm.
- The data structures found in specialized literature in MOO are:
  - a linear structure
    list, sorted or not
    $\rightarrow$ simple vs cost of the pairwise comparison
  - a tree structure
    AVL-Tree ($p = 2$), Quad-Tree ($p \geqslant 2$), ND-Tree ($p \geqslant 2$)
    $\rightarrow$ fast vs complexity for maintaining the structure

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ND-Tree

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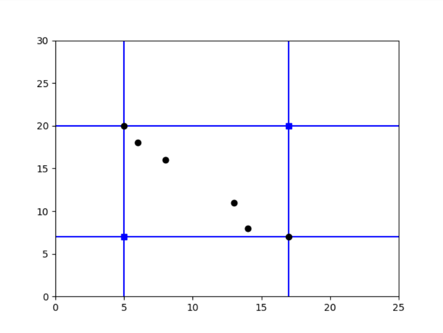

Principle:

- to divide the objective space into hyperrectangles
- if a hyperrectangle contains too many points then it is divided

Parameters:

σ: maximal number of points per hyperrectangle

δ: number of sub-hyperrectangles created when a hyperrectangle is divided

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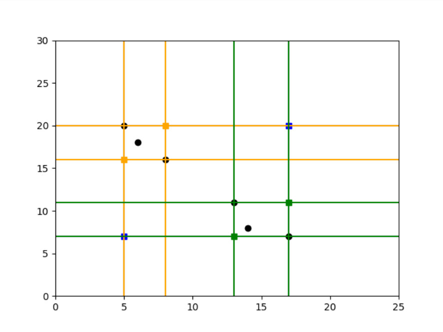

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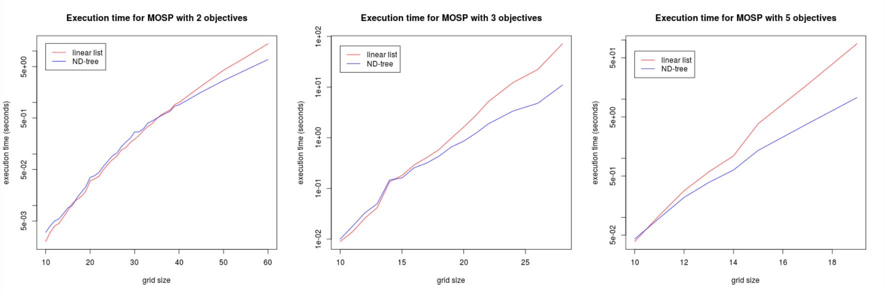
ND-Tree: numerical experiments

- One ND-Tree on each node of the graph:

Linear list vs ND-tree for a grid graph. Cost values are in $U[1; 100]$



X axis: Number of nodes (gridSize × gridsize) — Y axis: CPUt (in seconds) in logarithmic scale — Results in average on 50 runs

⇒ ND-tree appears competitive against the linear list when the number of objectives and the number of nodes are high

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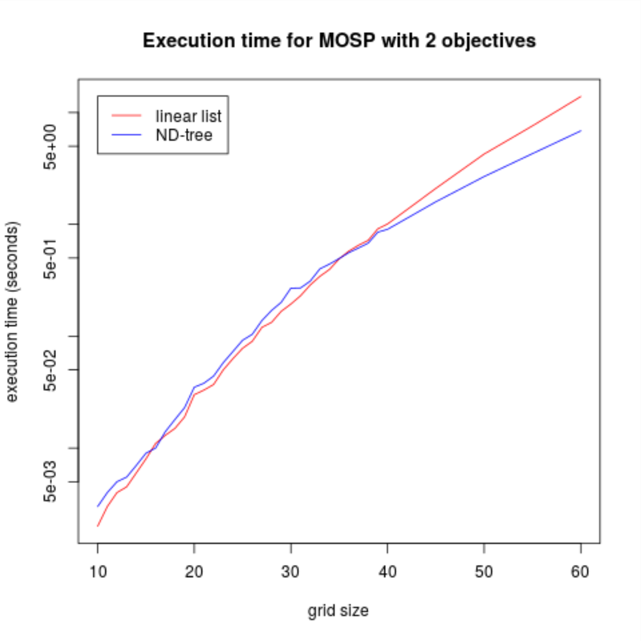

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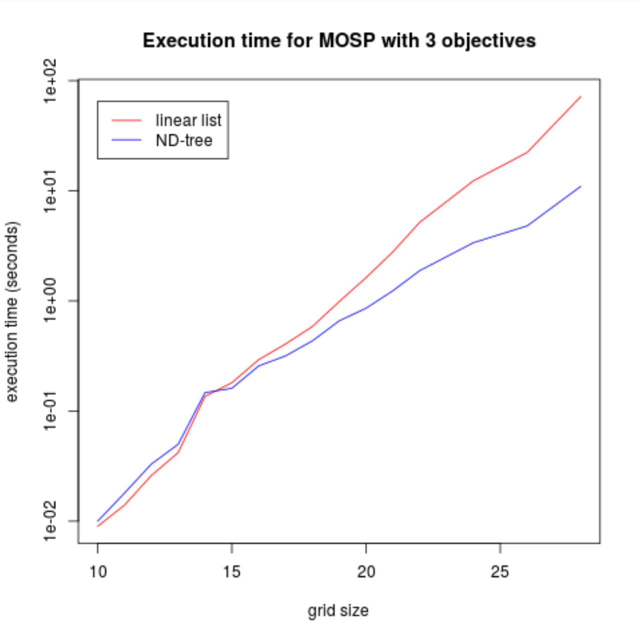

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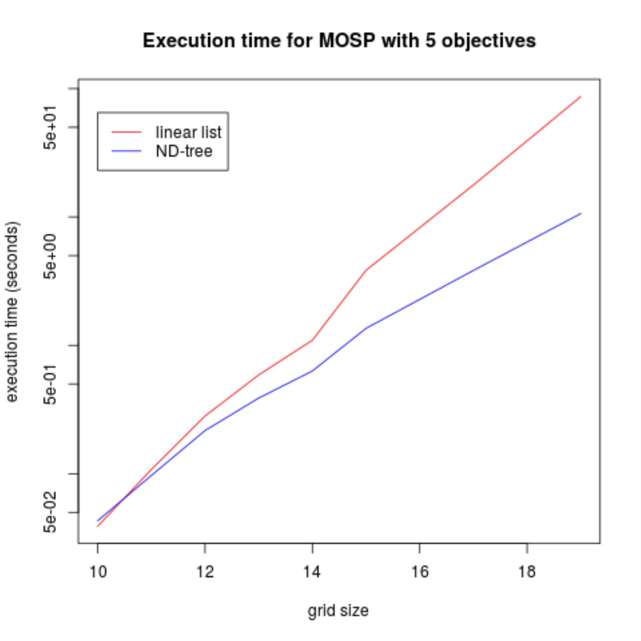

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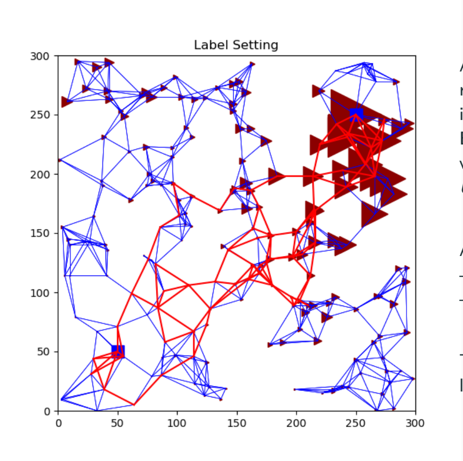
3. On the strategy of label propagation

Observation

- New assumption:
  1-to-1 (source s to destination t) efficient paths



A random graph with clusters (200 nodes randomly generated, each node is connected with its 4 closest neighbors according to the e Euclidean distance) and 3 linear objectives. Cost values are randomly selected in the range $U[1;300]$

All efficient shortest paths (in red) between
- the origin node (square in the south-west) and
- the destination node (square in the north-east)

The triangles represent the maximum number of labels on a node

Situation

- Number of labels may grow significantly near the termination node t.

- Alternative: a bi-directional strategy
  - Well-known in single objective case (Nicholson 1966; Pohl 1969)
  - Use two separated procedures:
    - a forward search from the origin node and
    - a backward search starting from the destination node

      $\rightarrow$ two search trees,
      potentially expanding fewer labels than a single search

- Existing literature in MOP:
  - Demeyer et al., 2013 (4OR journal)
    $p \geqslant 2$
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Speed-up techniques proposed in Demeyer et al., 2013

Main ideas:

- a forward and backward search (similar to the unidirectional algorithm)
- a stopping condition is based on the use of a vector of minimal values of objectives for temporary labels

Numerical experiments:

- up to 20 time faster for transportation graphs with 2 and 3 objectives (instances: sparse graphs representing transportation problems with hundreds of thousands nodes and links, average node degree between 2 and 3)
- mitigated for random graphs
- not pertinent for complete graphs and square grid graphs

⇒ performance of the strategy is dependant on the graph configuration and predicting the average speedup is difficult.

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Our ongoing attempt on the general case

- Observation: good results appear with a specific graph topology or when not considering a subset of X_E.
- Question: can we find a way to improve the computation time using a bi-directional strategy considering:
  - a (minimal) complete set of X_E
  - any graph topology
- Proposals :
  - a separation of the graph in two sub-graphs in preprocessing
  - identify graph topologies where a bi-directional separation strategy may be interesting or unnecessary

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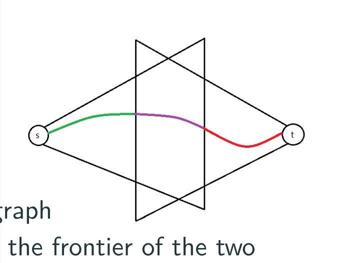
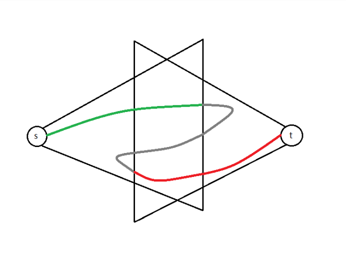
About the separation of the graph

Main ideas:

- compute a separation of the graph in two parts such that the source is in one sub-graph and the destination node is in the other one
- apply Martins' algorithm on each sub-graph
- merge the permanent labels existing at the frontier of the two sub-graphs





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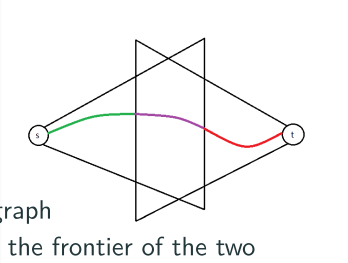

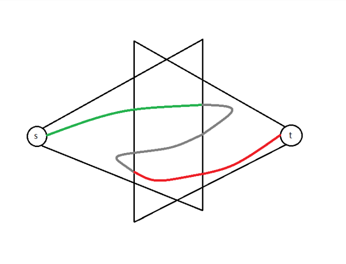

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- Question: is it possible to find a way to separate the graph such that there is no paths crossing each sub-graph twice or more?
    - *no* in general in an undirected graph
    - *maybe* in a directed graph
    - *yes* in a "well-oriented" graph

    - an example of "well-oriented" graph

- Remaining questions:
    - efficiency of a separation strategy in "well-oriented" graph?
    - can we find a way to compute a good separation in more random directed graphs?

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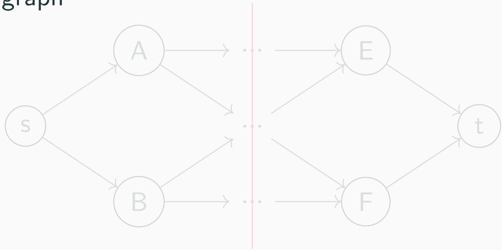

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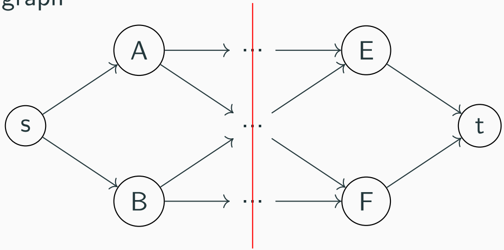

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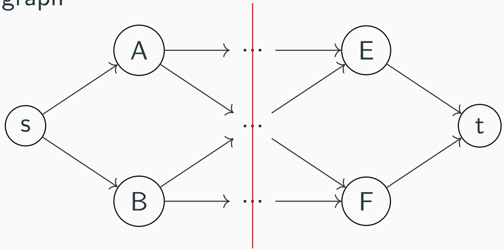

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4. Summary

Conclusion and ongoing works

→ A novel context of using ND-tree for maintaining temporary labels

→ A learning on the bi-directional strategy

→ A (coming) open-source package dealing with several MOSP

PROS:

- ND-tree: interesting even with few objectives
- bi-directional strategy: interesting for transportation graphs

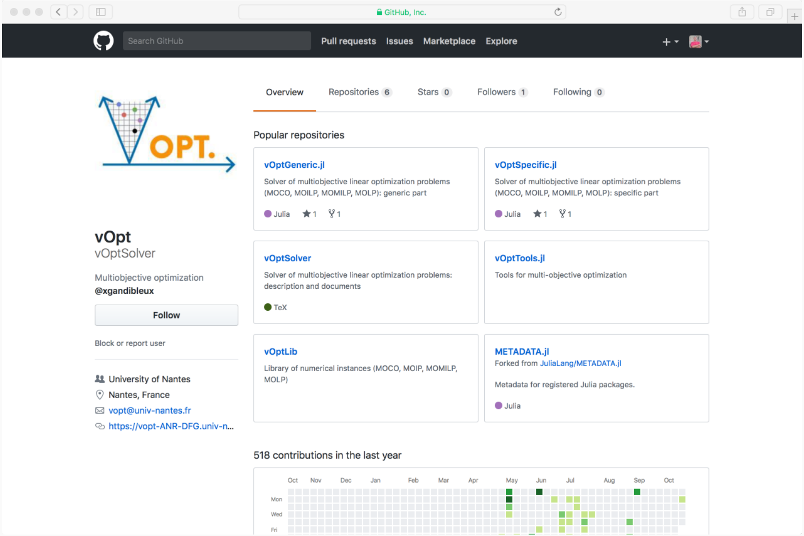
CONS:

- ND-tree: parameters to tune
- bi-directional strategy: predicting the average speedup is difficult

NOW:

- ND-tree: measuring the impact of σ and δ
- Bi-directional strategy: dealing with stated questions
- Label setting algorithm: working on others pending questions
- vOptSolver: releasing the `MOSP.jl` package

MOSP.jl awaited to be integrated to vOptSolver



http://voptsolver.github.io/vOptSolver/

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Homepage of vOptSolver:
`http://voptsolver.github.io/vOptSolver/`

Repository of vOptSolver:
`http://github.com/vOptSolver`

Contact concerning vOptSolver:
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