Linear programming for bi-objective stochastic logistics

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Outline

1. Bi-objective stochastic logistics
2. Tackling the bi-objective stochastic covering tour problem
3. Solving the bi-objective stochastic facility location problem
4. Conclusions and perspectives

Outline

Bi-objective stochastic logistics: an example

Disaster-relief situation

- Infrastructures are destroyed
- Supplies must be dispatched quickly
- Budget and fleet are limited
- It is not possible to visit every single village

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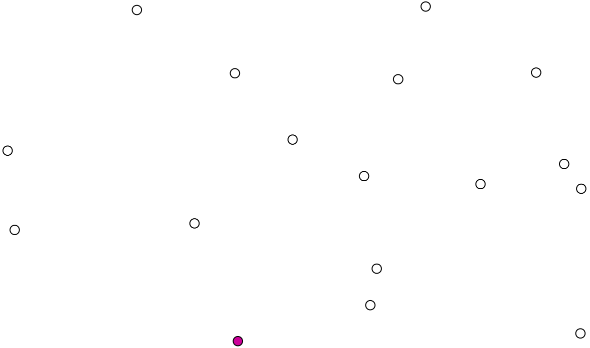
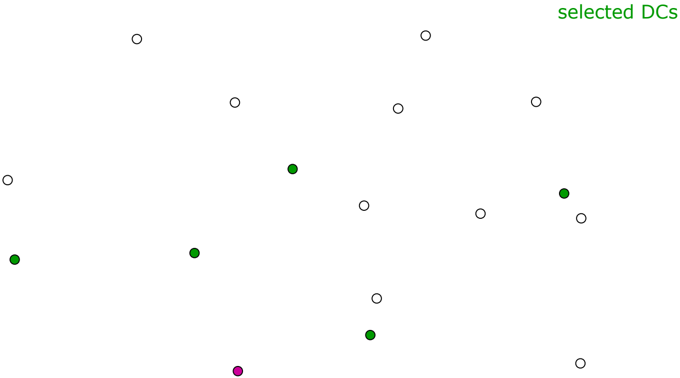
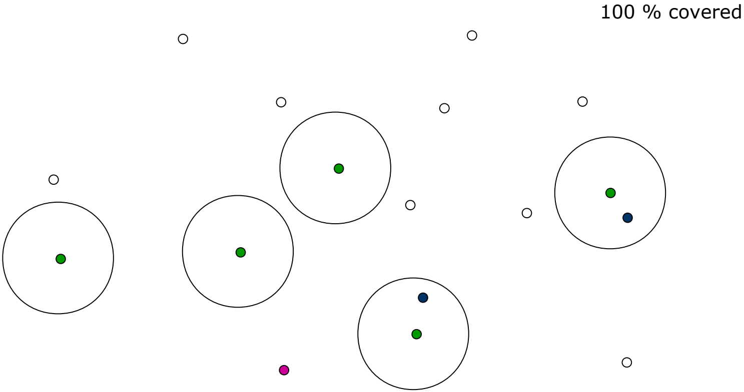
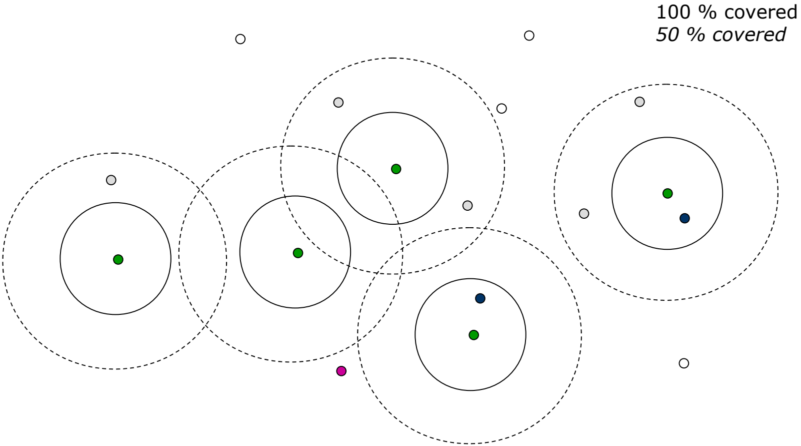
Distribution of supplies in such conditions

- Vehicles dispatch supplies to *Delivery Centers* (DCs)
- Villagers get supplies at the DCs
- Less villagers go to a DC if it is too far
- Additionally, demand is stochastic

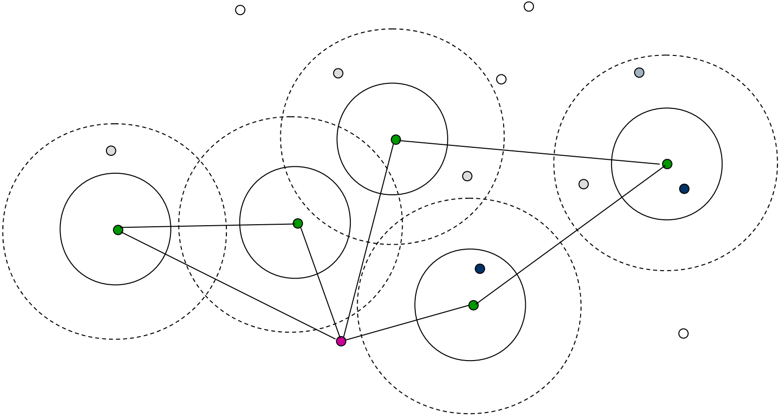


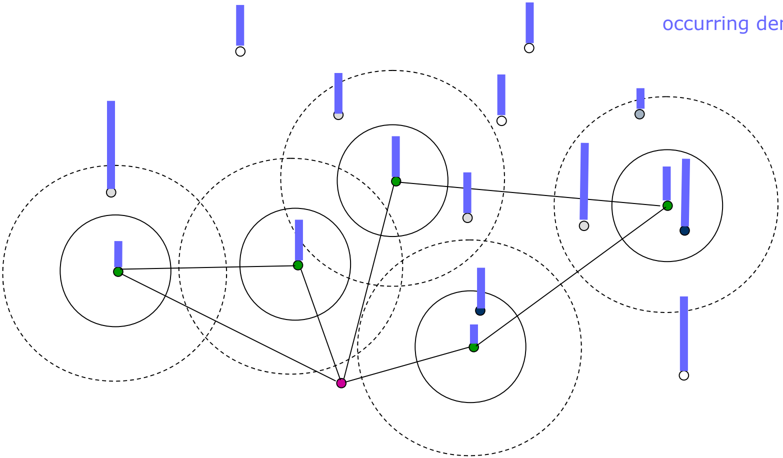

selected DCs

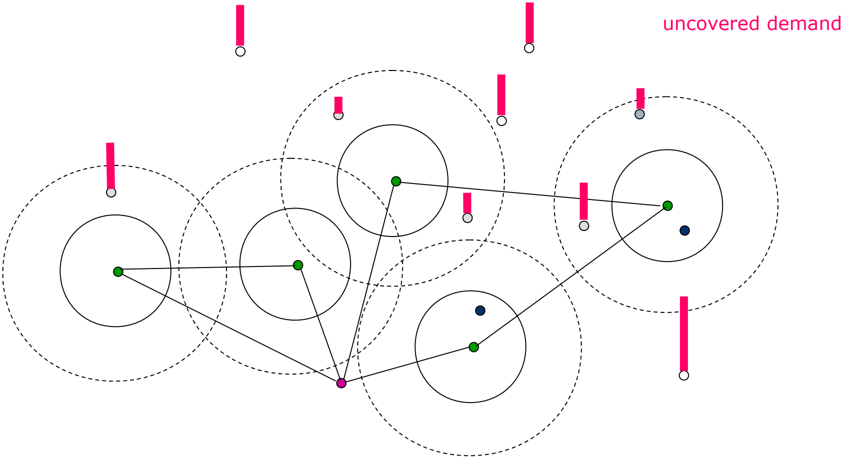

100 % covered


100 % covered
50 % covered

tours

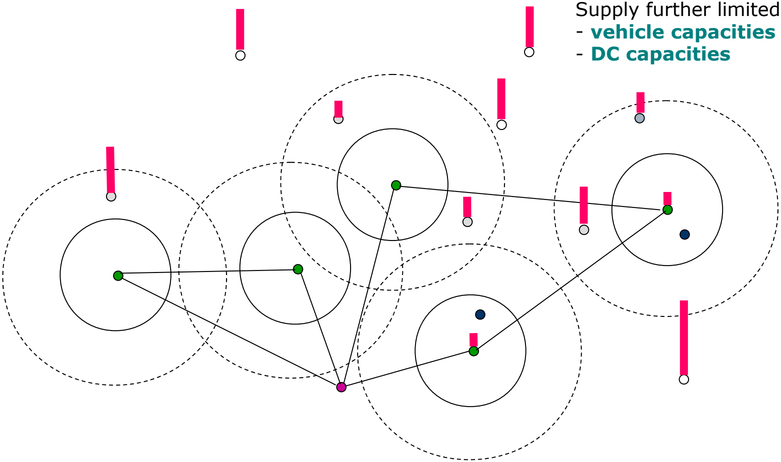



occurring demand


uncovered demand

Supply further limited by:

- **vehicle capacities**
- **DC capacities**



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Handling stochasticity

Sampling

- Consider N different samples
- Each sample corresponds to a possible realisation
- Variables related to stochasticity get a sample index
- This also increases the number of constraints
- Better approximations require more samples. . .

Solution approach

General framework: ϵ-constraint

Repeat the following steps:

1. $z_1 \leftarrow$ optimize f_1
2. Fix obtained tours
3. $z_2 \leftarrow$ optimize f_2
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Until no feasible solution is found

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Subtour elimination constraints are relaxed then generated dynamically in a branch-and-cut fashion.

(We also add some valid inequalities)

Experimental results

Instance	# villages	front size	CPU time (s)
Cherif Lo	10	13	11.77
Diender Guedj	22	34	>3d
Fandane	24	51	>3d
Fissel	21	107	>3d
Koul	16	29	569.11
Malicounda Wolof	12	27	36.08
Mbayene	12	31	81.75
Mboro	31	112	>3d
Meouane	17	31	231.13
Merina Dakhar	19	46	2451.19
Mont Roland	15	17	128.31
Ndiagagniao	24	92	>3d
Ndiakhene	9	6	1.42
Ndiass	18	55	6924.66
Ndieyene Shirak	11	27	49.21
Neugeniene	17	36	883.28
Ngandiouf	19	75	13261.8
Nguekhokh	19	36	573.29
Notto Gouye Diama	11	16	11.51
Notto	28	43	>3d
Pekesse	12	24	54.72
Pire Goureye	18	55	3663.46
Pout	29	55	>3d
Sandira	15	28	198.01
Tassette	21	33	4973.5
Thiadiaye	13	21	58.33
Thienaba	10	14	14.06
Thilmanka	14	39	268.81
Tiba Ndiaye	14	22	104.49
Touba Toul	20	43	12706.5

Fixed time limit: 8 hours

Experimental observations

- Each new solution takes longer than the previous one
- Upper bounds are quite accurate
- Lower bounds are quite poor

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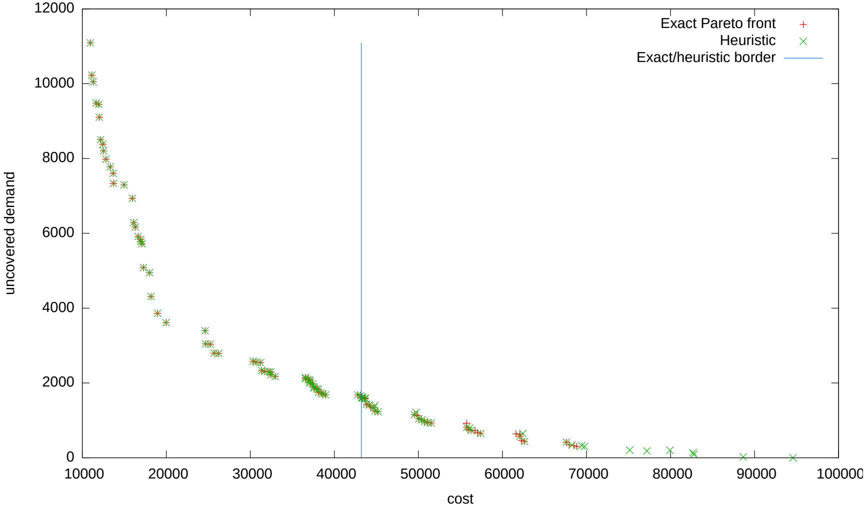
Heuristic ϵ-constraint

- Run the exact method for half the time budget
- After that, gradually increase the MIP gap tolerance
- 10 time slices of equal duration
- Increase tolerance by 10% in each slice

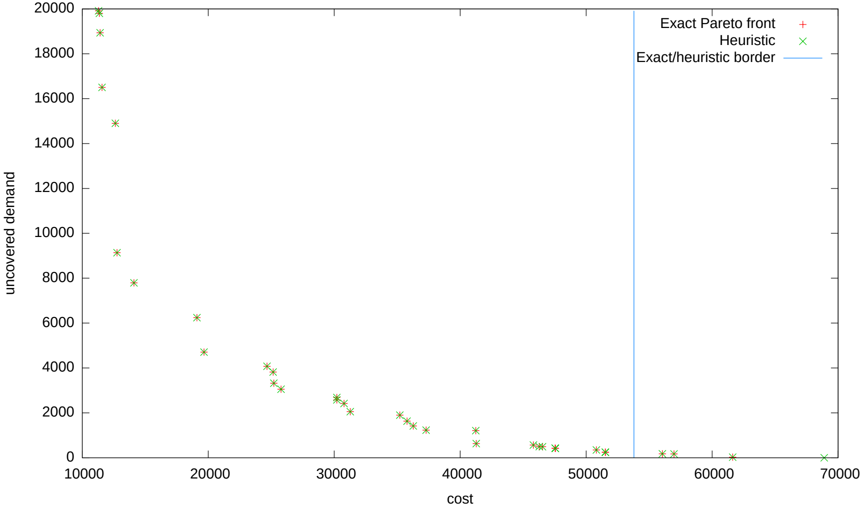
Heuristic ϵ-constraint: an example

24 villages



Heuristic ϵ-constraint: another example

22 villages



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Bi-objective stochastic facility location

A slightly different problem

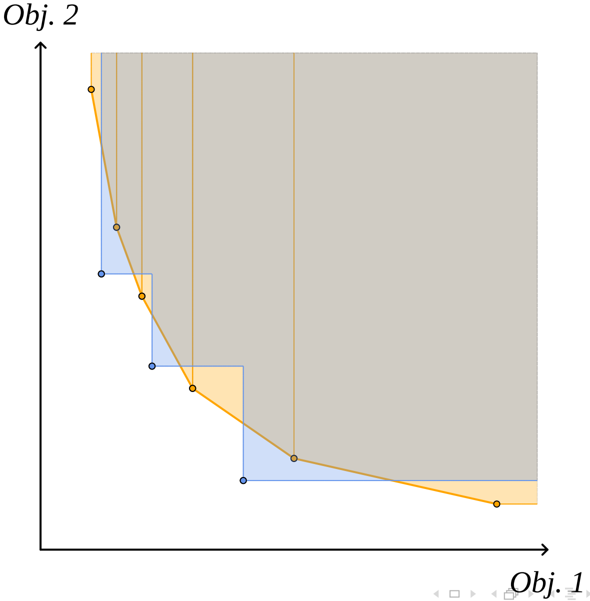
- In certain contexts routing can be disregarded:
  - Negligible routing costs
  - True facility location problems
- We then consider a bi-objective stochastic facility location problem
- The model is smaller and easier
- We still consider samples

Bi-objective branch-and-bound

General principles

- Lower bound set: continuous
  - Solve a succession of weighted-sum problems
- Upper bound set: discrete
  - Feasible solutions found during search
- Filtering continuous sets with discrete sets
- In one LB set, a given variable takes different values

Filtering and Branching on the objective space



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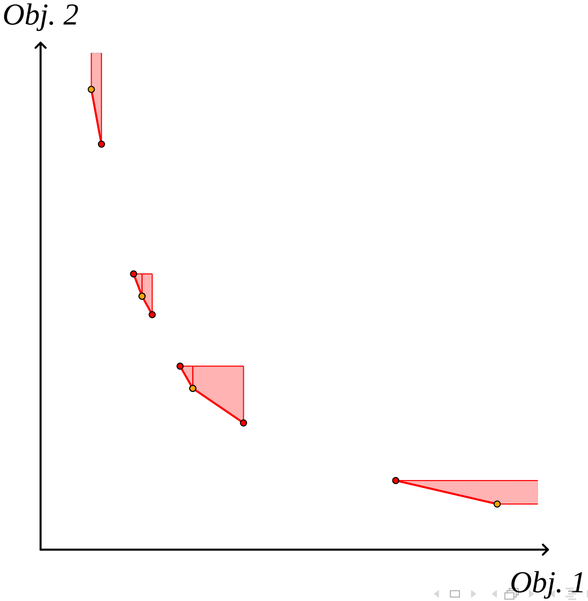
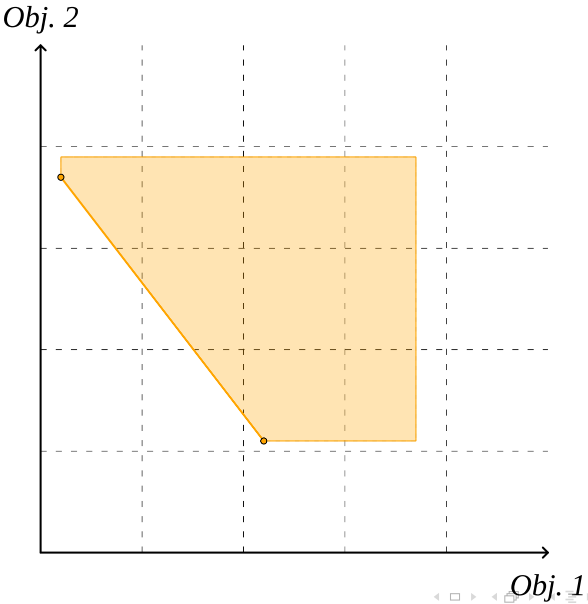


Tightening LB sets



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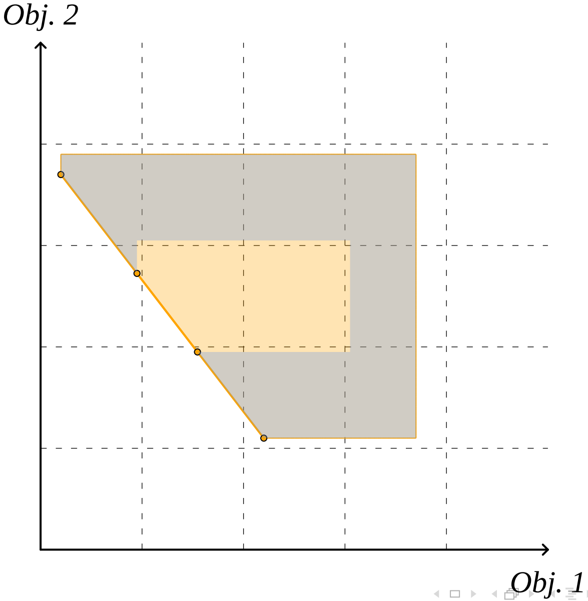
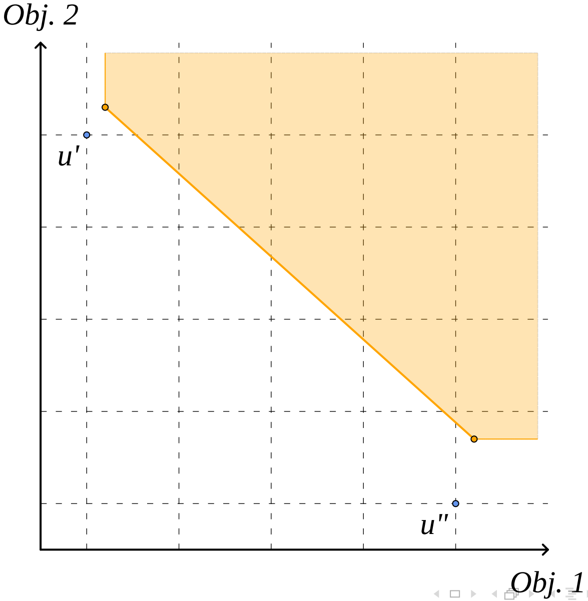


Improved filtering of the LB by the UB



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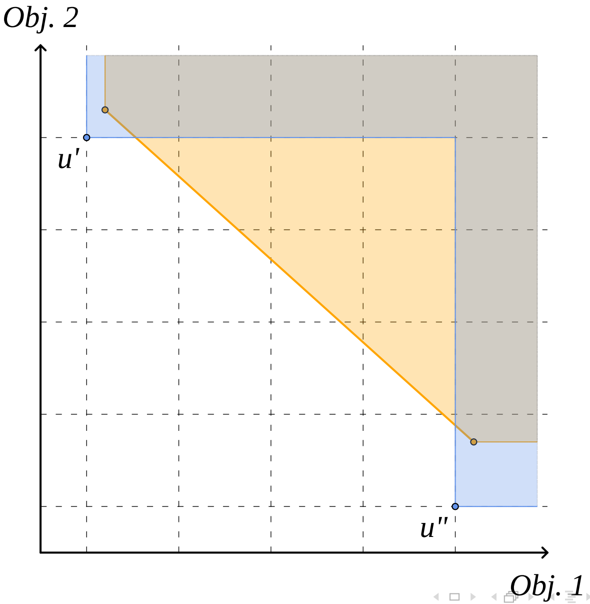

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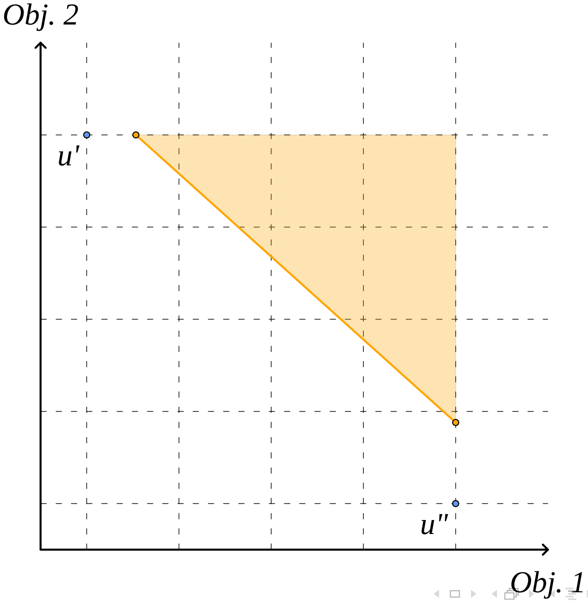

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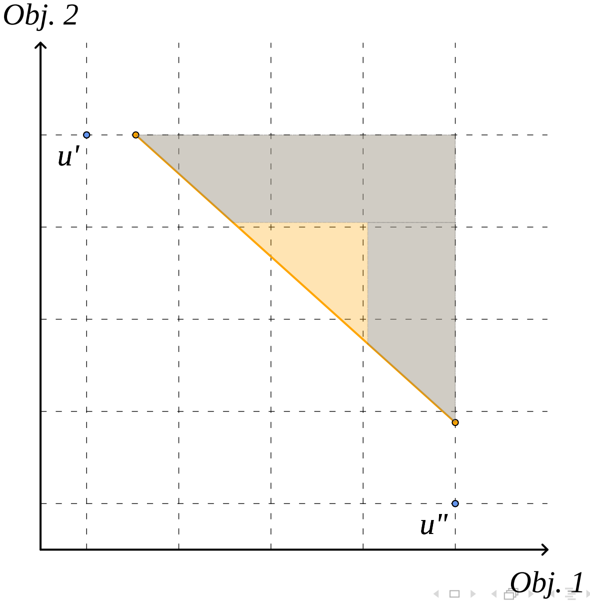
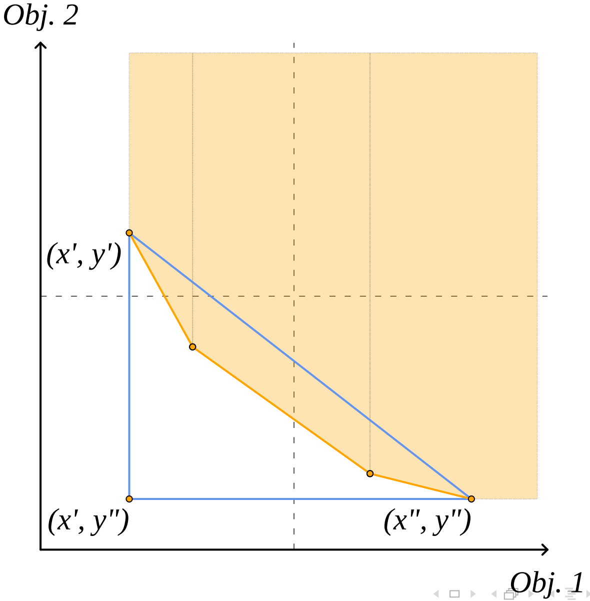


Speeding up the LB calculation



Preliminary results

n	ϵ-constraint	BIOBAB
1	00:00.06	00:00.05
21	00:00.50	00:00.22
44	00:03.35	00:01.06
55	00:05.23	00:01.50
72	00:11.31	00:02.97
90	00:19.57	00:04.72
106	00:25.16	00:07.09
120	00:35.20	00:10.21
132	00:44.24	00:12.17
134	00:49.04	00:13.87
135	00:51.24	00:13.13
162	01:14.99	00:29.56
182	02:01.92	00:55.14
203	02:26.15	00:58.33
226	03:18.26	01:26.79
254	04:39.49	01:27.80
264	05:18.96	01:45.88
266	05:16.44	01:42.70

n	ϵ-constraint	BIOBAB
275	05:43.66	02:45.56
294	07:05.64	03:56.28
295	07:58.92	03:55.53
296	07:31.77	02:24.54
326	10:17.07	03:08.51
342	11:34.00	04:41.01
355	11:55.75	04:59.68
370	13:26.78	10:10.52
388	14:56.58	09:44.29
399	16:47.61	12:33.67
410	17:04.94	08:11.64
428	19:12.50	09:27.39
436	19:53.31	09:29.75
449	22:02.28	18:08.80
458	23:04.22	16:12.70
472	26:55.21	15:02.00
482	27:58.90	15:48.09
499	31:29.57	25:58.03
500	30:36.79	19:56.05

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Conclusions and perspectives

Conclusions

- BIOBAB pros
  - Faster than ϵ-constraint
  - Better front approximation when interrupted
  - Potential for improvement (tree search, cutting planes, LB computation)
- BIOBAB cons
  - More effort to implement (for now)
  - Some numerical tweaking required

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Perspectives

- Consider more samples and see how both methods scale
- Use L-shaped method in order to keep models small

Thank you for your attention

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Mathematical model

First stage (1)

$$\min_{x,y,z}(f_1, f_2) \quad \text{s. t.} \tag{1}$$

$$f_1 = \tau \sum_{k\in K} \sum_{(i,j)\in E} d_{ij} x_{ijk} + \sum_{k\in K} \sum_{j\in V} c_j z_{jk} \tag{2}$$

$$f_2 = \mathbf{E}(R(y, z, \xi)) \tag{3}$$

$$\sum_{j\in V} y_{ij} = 1 \quad \forall i \in V \tag{4}$$

$$y_{ij} \leqslant \sum_{k\in K} z_{jk} \quad \forall i, j \in V \tag{5}$$

$$\sum_{j\in V} d_{ij} y_{ij} \leqslant d_{im} + M \left(1 - \sum_{k\in K} z_{mk}\right) \quad \forall i, m \in V \tag{6}$$

Mathematical model

First stage (2)

$$\sum_{k \in K} z_{jk} \leqslant 1 \quad \forall j \in V \tag{7}$$

$$\sum_{k \in K} z_{0k} = |K| \tag{8}$$

$$x_k(\delta(j)) = 2z_{jk} \quad \forall j \in V_0,\ k \in K \tag{9}$$

$$x_k(\delta(S)) \geqslant 2z_{jk} \quad \forall S \subseteq V,\, j \in S,\ k \in K \tag{10}$$

$$x_{ijk} \in \{0,1\} \quad \forall (i,j) \in E \setminus \delta(0), \quad x_{ijk} \in \{0,1,2\} \quad \forall (i,j) \in \delta(0) \tag{11}$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in V \tag{12}$$

$$z_{jk} \in \{0,1\} \quad \forall j \in V_0,\ k \in K \tag{13}$$

Mathematical model

Second stage

$$R(y,z,\xi) = \min_{u} \left[\sum_{i \in V} \xi_i w_i - \sum_{k \in K} \sum_{j \in V} u_{jk} \right] \quad \text{s. t.} \tag{14}$$

$$u_{jk} \leqslant \sum_{i \in V} \xi_i \, w_i \, \psi(d_{ij}) \, y_{ij} \quad \forall j \in V,\ k \in K \tag{15}$$

$$u_{jk} \leqslant \gamma_j z_{jk} \quad \forall j \in V,\ k \in K \tag{16}$$

$$\sum_{j \in V} u_{jk} \leqslant Q_k \quad \forall k \in K \tag{17}$$

$$u_{jk} \geqslant 0 \quad \forall j \in V,\ k \in K \tag{18}$$

Treatment of stochasticity: sampling

Consider N samples

$$f_2 = \frac{1}{N} \sum_{\nu=1}^{N} \left[\sum_{i \in V} \xi_i^{(\nu)} w_i - \sum_{k \in K} \sum_{j \in V} u_{jk}^{(\nu)} \right] \tag{19}$$

$$u_{jk}^{(\nu)} \leqslant \sum_{i \in V} \xi_i^{(\nu)} \, w_i \, \psi(d_{ij}) \, y_{ij} \quad \forall j \in V, \; k \in K, \; \nu \in 1..N \tag{20}$$

$$u_{jk}^{(\nu)} \leqslant \gamma_j z_{jk} \quad \forall j \in V, \; k \in K, \; \nu \in 1..N \tag{21}$$

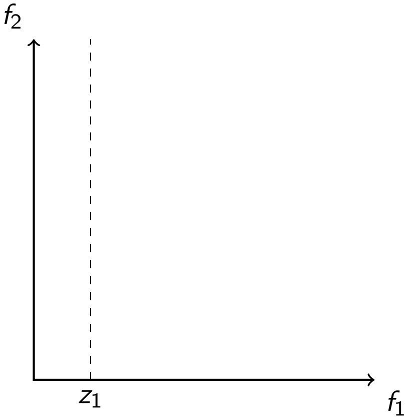
$$\sum_{j \in V} u_{jk}^{(\nu)} \leqslant Q_k \quad \forall k \in K, \; \nu \in 1..N \tag{22}$$

$$u_{jk}^{(\nu)} \geqslant 0 \quad \forall j \in V, \; k \in K, \; \nu \in 1..N \tag{23}$$

This change multiplies the number of required variables of the type u_{jk} by the factor N, as well as associated constraints.

The ϵ-constraint framework

1. $z_1 \leftarrow$ optimize f_1
2. Set obtained tours as constraints
3. $z_2 \leftarrow$ optimize f_2
4. Add ϵ-constraint: $f_2 \leqslant z_2 - \epsilon$



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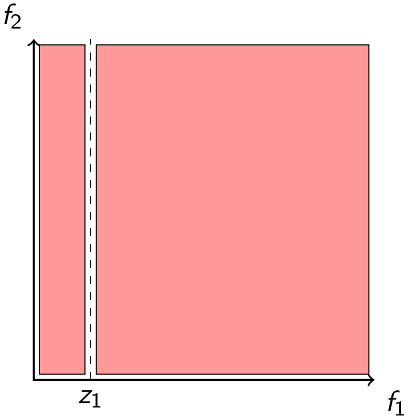

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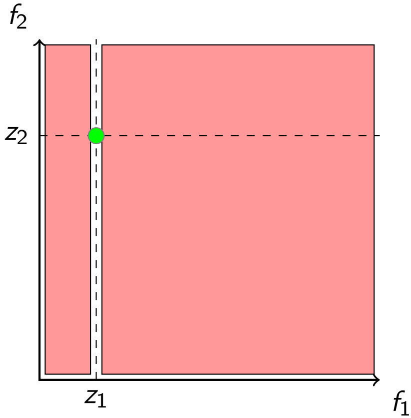

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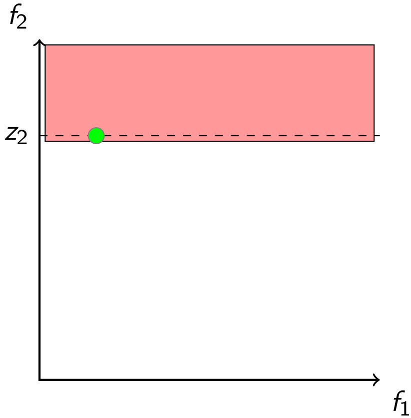

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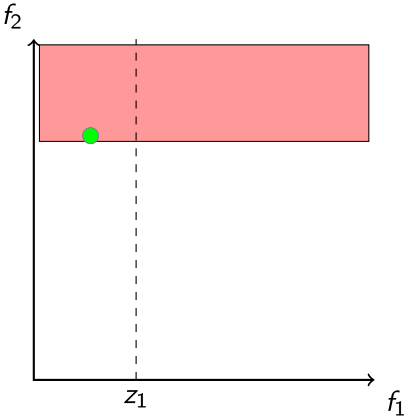

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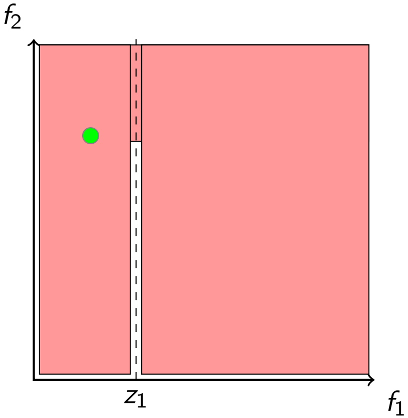

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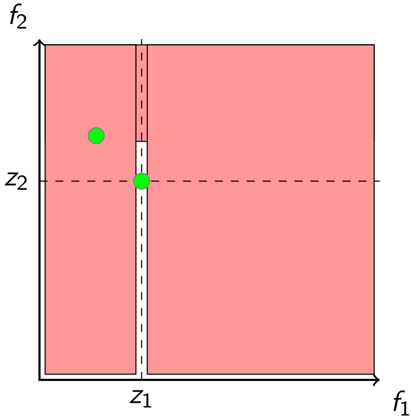

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