A branch-and-bound algorithm for convex multi-objective Mixed Integer Non-Linear Programming Problems

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Convex multi-objective MINLPs

$$
\begin{aligned}
\min f_k(x) \qquad & \forall k \in \{1, \ldots, p\} && (1)\\
g_i(x) \leq 0 \qquad & \forall i \in \{1, \ldots, m\} && (2)\\
x_j \in \mathbb{Z} \qquad & \forall j \in \{1, \ldots, r\} && (3)
\end{aligned}
$$

- n is the number of variables
- r is the number of general integer variables ($r \leq n$)
- $f_k, g_i : \mathbb{R}^n \to \mathbb{R}$ are twice continuously differentiable (non-linear) <u>convex</u> functions

Literature review

- Branch-and-bound algorithms for multi-objective (bi-objective) MILPs
  - Mavrotas and Diakoulaki 1998
  - Mavrotas and Diakoulaki 2005
  - Belotti, Soylu and Wiecek 2013
  - Vincent, Seipp, Ruzika, Przybylski and Gandibleux 2013
  - Parragh and Tricoire 2014
  - Stidsen, Andersed and Dammann 2014
- Multi-objective (bi-objective) NLPs
  - Fernández and Tóth 2007
  - Leyffer 2009
  - Ehrgott, Shao and Schöbel 2011

Branch-and-bound algorithm

- branching scheme
- dual bounds
- fathoming rules
- refinement procedure

Branch-and-bound algorithm

- Branching scheme:
  - At each level j of the decision tree, we generate one child node for each possible fixing of variable x_j to value l, with $l \in \{ub_j, \ldots, lb_j\}$
- Dual bounds:
  - The lower bound at the root node is computed by solving p single objective MINLP problems via a general-purpose MINLP solver.
  - At each node of the decision tree, the lower bound is computed by solving p single objective NLP problems obtained by relaxing integrality requirements and by taking into account the branching decisions up to the current node.

Fathoming rules

A node can be fathomed if:

- the corresponding relaxed problem is infeasible
- it is an integer feasible leaf node
- its lower bound is dominated by (at least) one of the solutions, say x^*, of the current Pareto set, i.e.,

$$LB_k \geq f_k(x^*) \quad \forall k \in \{1, \ldots, p\}$$

- each single objective NLP_k problem ($k \in \{1, \ldots, p\}$) is integer feasible and all the p integer solutions coincide

Starting Pareto set and solving leaf nodes

Weighted Sum method:

$$\begin{aligned} \min \sum_{k=1}^{p} \lambda_k f_k(x) & \\ g_i(x) \leq 0 & \qquad \forall i \in \{1, \ldots, m\} \\ x_j \in \mathbb{Z} & \qquad \forall j \in \{1, \ldots, r\}, \end{aligned}$$

with $0 \leq \lambda_k \leq 1 \; \forall k \in \{1, \ldots, p\}$ and $\sum_{k=1}^{p} \lambda_k = 1$.

Since we consider convex problems, the solution of the leaf nodes can generate all Pareto points by varying the weights (Censor 1977).

Refinement procedure

For each solution x^* in the current Pareto set Y^* and for each objective function $f_{\bar{k}}$ ($\bar{k} \in \{1, \ldots, p\}$), we solve the following model with $\tilde{f}_k$ set to $f_k(x^*)$.

$$
\begin{aligned}
\min f_{\bar{k}}(x) & \\
g_i(x) \leq 0 & \qquad \forall i \in \{1, \ldots, m\} \\
f_k(x) \leq \tilde{f}_k & \qquad \forall k \in \{1, \ldots, p\}, k \neq \bar{k} \\
x_j \in \mathbb{Z} & \qquad \forall j \in \{1, \ldots, r\}.
\end{aligned}
$$

Preliminary Computational experiments: Hydro Unit Commitment & Scheduling

A unit commitment problem of a generation company (Borghetti, D'Ambrosio, Lodi, Martello 2008):

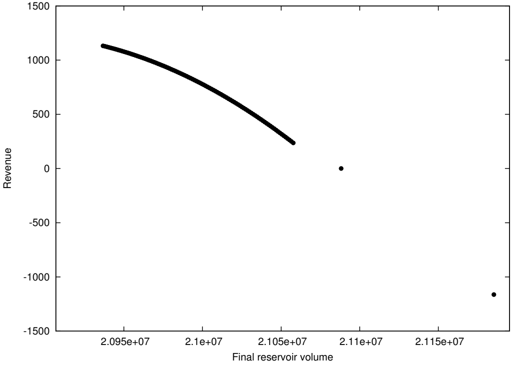
- find the optimal scheduling (maximize the power selling revenue) of a multiunit pump-storage hydro power station, for a short term period in which the electricity prices are forecast
- during the time horizon, a set of units can be:
  - used as turbines to produce power
  - used as pumps to pump water in the reservoir
  - switched off
- several physical and operational constraints are imposed
  - lower and upper bounds on the flows in the turbines
  - limits on the flow variations in two consecutive periods
  - water spillage to startup a pump or a turbine
  - in Borghetti et al. lower bound on the final reservoir volume

Hydro Unit Commitment & Scheduling: MINLP model

- Binary variables are used to model the units behavior
- Continuous variables model the water flow passing through turbines or pumped by pumps and the water volume in the reservoir
- Additional variables are used to model the physical and operational constraints
- Bi-objective model:
  - maximization of the revenue obtained from power selling: non-linear concave function
  - maximization of the reservoir volume at the end of the time horizon: linear function

A discontinuous Pareto set

Consider a single period of the time horizon and fix each of the 3 configurations (turbine on, pump on, both off): the Pareto set is the union of the three disjoint sets.



Characteristics of the instances

T: number of time periods of one hour considered in the instance

# T	# vars	# bin	# constr
1	18	8	19
2	30	14	34
3	42	20	49
4	54	26	64
5	66	32	79
6	78	38	94
7	90	44	109

Computational experiments: setting

- AMPL environment
- Intel Xeon 2.4 GHz with 8 GB Ram running Linux
- SCIP to solve single objective MINLPs
- Ipopt to solve single objective NLPs
- Weighted Sum method to obtain a starting Pareto set (step 0.1)
- Weighted Sum method to solve a leaf node (step 0.1)

Comparison

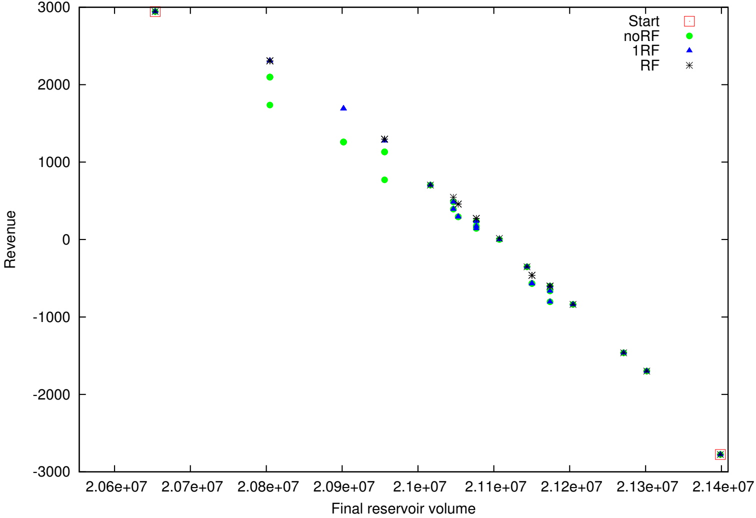
Comparison of three branch-and-bound versions:

- noRF: no refinement
- 1RF: refinement procedure only executed at the end of the resolution
- RF: refinement procedure executed at each update of the Pareto set

Comparison of three branch-and-bound versions

	Number of solutions			CPU time (sec)		
# T	noRF	1RF	RF	noRF	1RF	RF
1	4	4	4	1	1	1
2	11	11	11	3	3	3
3	35	35	30	12	12	15
4	61	61	49	43	43	57
5	108	108	79	150	150	229
6	179	179	120	534	534	891
7	257	257	134	1946	1948	3861

Pareto sets of the three branch-and-bound versions



Fathoming statistics

# T	# nodes	# dom	# leaf
1	12	1	1
2	55	1	5
3	233	4	19
4	862	11	65
5	3056	26	211
6	10415	54	665
7	34185	175	1995

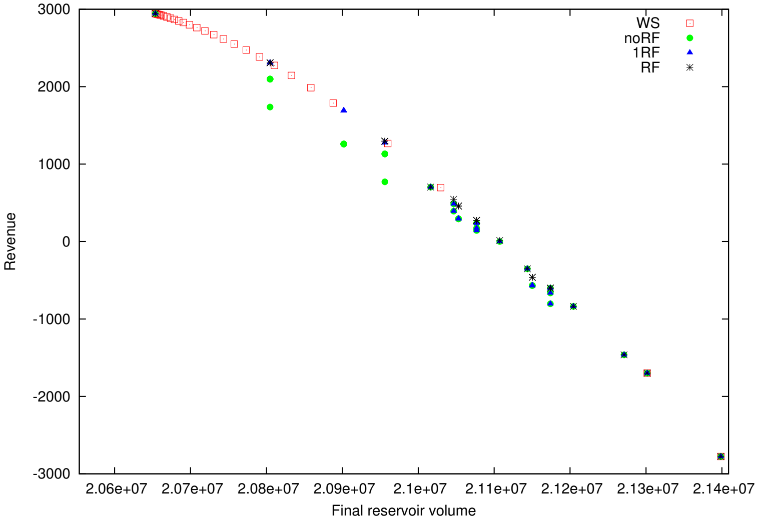
Comparison with the Weighted Sum method (T=3)

The Weighted Sum method:

- was executed with a step of 0.001, i.e. executed for 1000 iterations
- ended up in obtaining 27 solutions
- solutions are characterized by a high revenue and a limited final reservoir

The branch-and-bound algorithm derives a more diverse Pareto set. The RF solutions are characterized by solutions having revenue and volume in wider ranges.

Comparison with the Weighted Sum method (T=3)



Conclusions

- We have presented a branch-and-bound algorithm for convex multi-objective MINLPs
- Preliminary computational experiments on instances of Hydro Unit Commitment & Scheduling show that the method finds a more diverse Pareto set compared to Weighted Sum method

- Future research will be devoted to
  - compare the proposed method to the ϵ-constraint method
  - improve the way of solving the leaf nodes and the fathoming rules to speed up the overall solution process
  - test additional instances (e.g. convex nonlinear Knapsack Problem)

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Thank you for your attention!